VALUATION MODELS FOR CONVERTIBLE MORTGAGES

By

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The objective of this study is the development of models for the valuation of convertible mortgages. Two financial techniques, discounted cash flow analysis and option pricing theory, are used to value convertible mortgages. Valuation models are developed for both of these techniques, and the appropriateness of each method used to value the convertible is discussed.

A questionnaire was mailed to a wide range of large institutional investors. Data from the questionnaire provide an insight to industry perception of convertible mortgages, the common features of the instruments being written, and an indication of how these mortgages are currently being priced. The gathered data are used to determine the appropriate characteristics of a representative convertible mortgage.

Computer programs are written for the discounted cash flow model and option pricing theory model. The discounted cash flow model allows a user to incorporate Monte Carlo simulation into the valuation process. Each model is then used to evaluate what is believed to be a representative convertible mortgage. Sensitivity analysis is also applied to both models to test a range of input variables.

The value of the representative mortgage is found to be similar for both models. However, value estimates for the component cash flows of the convertible mortgage (interest income, participation income, and conversion value income) are not very similar between the models. Reasons for the variation in component cash flows are discussed.

The appropriateness of the models could not be evaluated on an empirical basis due to lack of data. Consequently, the evaluation had to be carried out by an analysis of the important features or characteristics of the convertible mortgage. Both models appear to provide plausible approximations of value. It is believed that either model can assist investors in making better investment decisions.

CHAPTER ONE
INTRODUCTION

Background

Some domestic and foreign investors have, in the last several years, placed a premium on quality real estate. This increased demand, coupled with a relatively finite supply of quality properties and increased rentals from tenants, have caused values to appreciate relative to other real estate. Many of these properties are owned by major institutions, large corporate users, or substantial private families who are not active sellers. These owners recognize the difficulties in replacing their prime real estate with an investment of equal quality. The first-tier properties that do come on the market are usually in such demand that the acquisition price is increased to the point where the initial cash-on-cash yield dips far below levels that would be acceptable for similar risk investments. Investors in these properties might not expect to receive a normal return on invested capital in today's market for at least three to five years, when a substantial portion of the building's leases would renew at the then prevailing market rates.

The competitive investment climate that exists for prime real estate has generated interest in creative

financing methods. Today an assortment of financial instruments exist which can provide lenders acceptable income yields immediately or provide protection from interest rate risk. One such method is the convertible mortgage which is a debt instrument that has an equity conversion option. As a hybrid financing vehicle, the convertible mortgage offers the borrower a below-market interest rate while giving the lender the option of converting the mortgage into equity ownership. The mortgage provides the borrower (typically a developer) with structured take-out financing, usually interest-only, at 200-400 basis points below market rates, with a due date of not less than five years. The lender has several conversion options; it can call the loan, renew at specified terms, or assume 100-percent ownership by converting the unpaid balance into equity and buying the borrower's equity position at a fixed price or a price based upon some predetermined formula. A typical convertible mortgage is quite often structured as a participating loan with a discounted coupon rate that includes additional income payments to the lender in the form of cash flow participation.

The convertible mortgage combines the features of traditional mortgage debt and of equity ownership into a single instrument that offers benefits to both parties to the loan arrangement. There are several reasons why lenders find convertible mortgages attractive. Lenders have the creditor priority of secured debt and immediate fixed income.

They will receive a considerably higher return even at subsidized interest levels, compared to the initial yields available from an outright purchase. Lenders also have an option to purchase the property, at a fixed price based upon current valuation, which should provide a hedge against unanticipated inflation. Finally, the lender (buyer) has an opportunity during the period of the loan to verify the developer's income and expense projections with actual audited statements.

Borrowers/developers find convertible mortgages attractive because they receive a favorable interest rate. In addition, the loan-to-value ratio is higher, so they can obtain a greater amount of financing and invest less of their own equity capital. The borrowers usually retain full property management until conversion occurs. Perhaps most importantly, the developers have the opportunity to exhaust most of the available tax benefits associated with the property before the conversion occurs. Allowing the tax benefits to be utilized through the use of a convertible mortgage is a key element in persuading the developer to sell his project on a forward funding basis. The developer can take advantage of depreciation deductions that might otherwise be unusable by certain tax-exempt investors such as pension funds. The developer has the added bonus of converting ordinary income from a base year sale to a capital

gain, since the purchase option will be exercised safely beyond the required six month holding period.

Perhaps the most compelling incentive for the developer to sell his project is that he will receive approximately 75 percent or more of the agreed upon value of the property at the completion and lease-up of the project when the convertible mortgage loan is received. The developer will pay no tax on receipt of these funds because they are considered refinance proceeds which are used to replace a short term construction loan. The balance of the purchase price would normally be received at the close of the sale, prompted by the exercise of the purchase option.

Research Question

Lenders pay for the conversion privilege by accepting a lower interest rate on convertible mortgage loans. Intuitively, the cost to the lender for the conversion right is the present value of future lost interest payments. However, the value of the conversion option may not equal the present value of lost interest payments. Lost interest payments relate to cost, whereas conversion value relates to future property value.

The primary purpose of this paper is to develop methods for valuing the conversion right using current financial theory. This study also identifies the users of these unique hybrid instruments and examines how and why they are used. It also examines methods used to value convertible mortgages.

Scope and Limitations of Research

This research will attempt to offer convertible mortgage investors an alternative, and perhaps more sophisticated, approach to valuation of these securities. A range of value can be forecast by the models for these mortgages based upon the application of reasonable assumptions for unknown variables. Sufficient data on actual convertible mortgages are not readily available, however; therefore, this study will not be able to conclude whether these instruments are being mispriced.

The information about actual convertibles which was obtained through a questionnaire to institutional investors has been tested against the models developed. Testing requires that assumptions be made about unknown variable values associated with actual convertible mortgages. Results of this testing indicate that pricing appears to be reasonable based upon the assumptions made.

Significance and Application

The theoretical and practical significance of this study may be considerable. There are at least two practical reasons why the value of the conversion privilege should be determined. First, the original interest rate of the loan is directly correlated with future conversion expectations. To establish a proper coupon rate, there must be a determination of how many basis points can be given up for the conversion option. Secondly, since the option to purchase can be sold

after origination, its value may need to be determined. If an accurate financial valuation model can be developed, the instruments can be more easily priced and perhaps more readily used.

A discussion of the instrument's legal characteristics and projected cash flows can provide clues to the proper theoretical approach to convertible mortgage valuation. It is evident that current financial theory, particularly in the areas of discounted cash flow analysis and option pricing techniques, is central to the valuation and understanding of this type of mortgage. After reviewing various valuation techniques, computer models are developed to arrive at convertible mortgage values.

Organization of the Study

This study is developed in three stages. The first portion of the research deals with application of current financial techniques to the development of convertible mortgage valuation models. Specifically, discounted cash flow analysis is considered as an approach to valuation. A convertible mortgage valuation model was developed to provide pertinent information for decision-making purposes. The model allows users to incorporate sophisticated risk analysis into the process.

Option pricing theory also appears to be useful in valuing convertible mortgages. A convertible mortgage can be viewed as a straight mortgage with an option attached.

Therefore a valuation model based on option pricing techniques is developed.

The second major portion of the study deals with the market for convertible mortgages. A questionnaire is developed to gather convertible mortgage data from a wide range of large institutional investors. The results of the questionnaire study provide a better understanding of how and why convertible mortgages are used. The survey responses will provide insights into the industry's perception of convertibles, the common features of the instruments being written, and an indication of how coupon discounts are established. Data are also gathered on actual convertible mortgage contracts that the institutions write.

A logical third and final step to the study is testing the valuation models against the data on convertible mortgages that are provided by the questionnaire responses. This provides information on the applicability of the valuation models in setting market prices of convertibles.

CHAPTER TWO
REVIEW OF THE LITERATURE

Convertible Mortgage Literature

Forty years ago, a major innovation that significantly altered the basic mortgage instrument was the introduction of the amortized loan. This development was brought about in the 1940s by the creation of the Federal Housing Administration and the insurance industry's establishment of a consistent policy of mortgage amortization as a result of the bitter lessons it had learned in the Great Depression. Without any question, the amortized mortgage has provided a myriad of benefits to everyone involved in real estate investment.

In recent times, particularly during the decade of the 1970s, inflation and its effect on the "standard" mortgage instrument have caused problems for both lenders and borrowers. Lenders systematically estimated interest rates to be lower than they would be if inflation was fully anticipated and "built in" to those rates. Competition for savers' dollars was intense, and financial institutions were forced to pay high returns to savers. Lenders held low yielding fixed-rate loan portfolios, while at the same time

they were required to pay high rates on deposits. As inflation intensified, borrowers faced prohibitively high rates on new real estate loans. In an attempt to deal with the problems that both lenders and borrowers face with the standard fixed-rate mortgage instrument in periods of significant inflation, many alternatives were proposed as potential solutions. Some of these alternatives, such as graduated payment mortgages and variable interest-rate mortgages, have been adopted at both the federal and state level by regulatory agencies governing the investment policies of lending institutions. Several other instruments have been proposed and tested with some success.

The convertible mortgage is one of many alternative mortgage instruments that lenders use to reduce interest-rate risk. A review of the literature specifically related to convertible mortgages shows a lack of useful information regarding valuation of these hybrid securities. Various articles, written primarily by mortgage-financing consultants and brokers, describe the typical convertible mortgage, suggest the instrument's advantages and disadvantages, and discuss legal problems associated with this type of mortgage (Jones, 1981; Oharenko, 1984; Strawn, 1982; Vitt and Bernstein, 1976; Vitt, 1975; White and Wiest, 1984).

The articles do not attempt to value the conversion option. Vitt and Oharenko calculate yields to the lender, but there is no attempt to partition this yield between

interest income and appreciation income. The value of the conversion option is qualitatively defined, but never quantitatively expressed. The technique used to determine an appropriate coupon rate is not explained. How do lenders price convertible mortgages when they are originated, and what would the mortgage be worth if the lenders wished to sell it? These are questions left unanswered in the literature on convertible mortgages.

Valuation Techniques

Since the existing literature on convertible mortgages does not provide clues for proper valuation, one must turn to parallels in the literature on valuation of corporate securities. Fortunately, extensive research has been carried out regarding the valuation of other securities, and academic literature is voluminous on this subject.

Corporate bonds that are convertible are of particular interest to this study, because these bonds share many characteristics that make them similar to convertible mortgages. Although research in the area of convertible bonds is less plentiful than research of common stocks and straight bonds, there has been sufficient work in this area to provide useful techniques for the valuation of convertible mortgages. While convertible bonds and convertible mortgages are not identical instruments, they share enough common features to allow the extension of theory from one security to the other.

The literature provides two approaches to convertible bond valuation. Discounted cash flow analysis is a standard technique in security analysis and real estate, and this method has been applied to convertible bond analysis for quite some time. A second approach to convertible bond valuation, and therefore convertible mortgage appraisal, is found in the theory of option pricing.

The Discounted Cash Flow Literature

Finding the present value of an investment security using discounted cash flow analysis is a well established practice in finance. Eugene F. Brigham summarizes a chapter in Financial Management Theory and Practice titled "Stock and Bond Values" as follows: "In all cases, security values were found to be the present value of the future cash flows expected from the security" (1979, p. 89). The value of securities should be calculated as the present value of their future cash flows; however, estimating what the future cash flows will be is a difficult task.

Pricing a convertible bond is considerably more complex than pricing a straight bond. The complications result from the unpredictability of future stock prices (and consequently the conversion value) and from the relationship between the conversion value and the investment value. If the convertible is callable, the pricing is complicated further.

Williams and Findlay (1974) revise the straight bond valuation model in order to value a convertible bond. The following is their straight bond valuation formula:

$$P_O = \sum_{t=1}^{n} \frac{I_t}{(1+i)^t} + \frac{P_n}{(1+i)^n} \qquad (2.1)$$

where

P_O = current value of the bond

I_t = annual dollar coupon paid on the bond

P_n = par value of the bond

i = yield to maturity of the bond

n = number of years to maturity

In the case of the convertible, the time horizon for holding the bond will be shorter than the number of years to maturity if conversion occurs. A revised equation taking conversion into account would be

$$P_O = \sum_{t=1}^{N} \frac{I_t}{(1+k)^t} + \frac{TV}{(1+k)^N} \qquad (2.2)$$

where

TV = terminal value of the convertible

k = expected rate of return from holding the convertible

N = expected holding period ($N<n$)

There are difficulties in using this model. First, how does one estimate N, the expected holding period? Second, how is the terminal value (TV) of the bond determined?

To overcome the problems just mentioned, Brigham developed a model for valuing convertible bonds, based upon conversion and straight bond values (Brigham, 1966). A bond's value depends upon the number of years to expiration, the anticipated movement in interest rates, and corporate risk. This is true because the market value of a bond is a function of time to expiration. Brigham's model assumes no change in market interest rates or risk over time. Conversion value depends upon investors' expectations of future stock value, which is stated in terms of the price of the stock when the convertible was issued, together with an assumed constant growth factor.

Market price of the convertible is portrayed by Brigham as intersecting conversion value at some time m, before the bond matures. Given any record of the firm's policy in calling convertibles and its expected common-stock growth rate, the investor can estimate the date of conversion m. The value of the convertible would then be determined by

$$P_O = \sum_{t=1}^{m} \frac{I_t}{(1+k)^t} + \frac{C_m}{(1+k)^m} \qquad (2.3)$$

where

C_m = value of convertible when called[1]

$$= Z\left(\frac{P_n}{S_e}\right)(1+g)^m$$

S_e = conversion price of stock

g = annual growth rate

Z = price of stock when convertible was issued

m = estimated date of conversion

The critical estimate in the Brigham model is the expected stock price at time m, which is determined by growth rate g. The difficulty of determining C_m and m, as well as the restrictive nature of some of the model's assumptions, has been noted by others (Van Horne, 1974).

Baumol, Malkiel, and Quandt (1966) constructed a convertible valuation model in much the same manner as Brigham, and their model is also dependent upon investors' expectations of future common-stock prices. Baumol, Malkiel, and Quandt are vague as to the conversion date, as well as the role that the firm plays in determining this date by exercising the call option. On the other hand, they discuss the appropriate discount factor in greater detail than did Brigham. They suggest that this rate should be the yield to

maturity if there is no premium over bond value; if there is no premium over conversion value, the investor's expected rate of return on equity is the proper discount factor. If the market value exceeds both bond value and conversion value, some rate between these two would be appropriate.

Based upon the variables developed in the Brigham and Baumol, Malkiel, and Quandt models, the convertible-bond valuation model is a simple extension of the analytical models used to value common stocks, preferred stock, or bonds. The major difficulty in using the convertible bond model arises from estimating values for additional variables that are not relevant in the case of simpler securities.

At any point in time prior to call, the investor holding a convertible bond can expect to receive the higher of conversion value or investment value. Which value is received will depend upon the value of the common stock. Poensgen (1966) incorporates probability theory into convertible bond modeling and suggests that the expected market value of a convertible bond can be derived as follows:

$$P_O = y \int_o^Y \left(\frac{P}{Y}\right) dZ + \int_Y^\infty P \left(\frac{P}{Y}\right) dP \qquad (2.4)$$

where

y = investment value and the other variables are as defined before

$\left(\frac{P}{Y}\right)$ = conditional probability

= probability of P occurring given that Y has occurred.

J. Walter and A. Que (1973) attempted to improve on the conventional model developed by Poensgen (1966) by using Monte Carlo simulation to forecast rates of return on convertible bonds, conditional upon the simulated behavior of the underlying stock. They concluded:

> Behavioral input derived for the simulation model attested to the powerful influence of the relationship between conversion values and straight bond values upon convertible bond premiums and to the asymmetry of premiums, depending on whether conversion values or straight bond values dominated. (Walter and Que, 1973, p. 730)

The literature shows that discounted cash flow analysis has played a role in the valuation of convertible bonds. This technique has not, however, been considered totally adequate in determining convertible bond values. Given the similarities between convertible bonds and convertible mortgages, it seems likely that this technique can be applied to the valuation of the conversion right associated with a convertible mortgage. In fact, one of the variables that impacts on value and presents problems to convertible bond valuation should not create a problem in mortgage valuation.

Convertible mortgages are not callable, a feature which simplifies the attempt to value these securities.

Option Pricing Literature

The intent of this paper is to develop a technique for estimating the value of the conversion right in a typical convertible mortgage. Building upon previous research in the area of option pricing theory, it is possible to derive a pricing formula for the valuation of convertible mortgage conversions rights.

Black-Scholes Model

In a seminal paper, F. Black and M. Scholes (1973) presented a complete general equilibrium theory of option pricing. Until their work, no satisfactory analytical formula existed to determine the value of stock options. Their work was particularly attractive because the final formula is a function of observable variables: the stock price, the exercise price, the time to maturity, the interest rate, and the volatility of the stock. Black and Scholes state that the analytical method used in determining the formula can be applied to virtually any financial security. It thus opened up a new era in the pricing of financial instruments.

Specifically, Black and Scholes developed a model for estimating the price of a European call option on the basis of its term to expiration and the relationship between the price of the stock and the striking price of the option. The Black-Scholes formula is

$$P_C = P_S N(d_1) - P_E e^{-R_F T} N(d_2) \qquad (2.4)$$

where

P_C = price of call option

P_S = price of the stock

P_E = striking price of the option

R_F = continuously compounded interest rate per time period

e = 2.71228, ...

T = number of time periods to expiration

$N(d_1)$ and $N(d_2)$ = the values of the cumulative normal distribution, defined by the following expressions:

$$d_1 = \frac{\ln(P_S/P_E) + (R_F + \alpha^2/2)\, T}{\alpha\sqrt{T}}$$

$$d_2 = \frac{\ln(P_S/P_E) + (R_F - \alpha^2/2)\, T}{\alpha\sqrt{T}}$$

and α^2 is the variance of a continuously compounded rate of return on the stock per time period.

Estimated option prices vary directly with an option's term to maturity and with the difference between the stock's market price and the option's striking price. The stock option value also increases with the variance of the continuously compounded rate of return on the stock price, reflecting the logic that greater volatility increases the chance that the option will become more valuable.

Extension of the Black-Scholes Model

The Black-Scholes formula is applicable only under rather restrictive assumptions.[2] Further development of the formula has included research to relax many of these assumptions. Specifically, modifications have been made to measure the effects of taxes, dividends, and variable interest rates, on option prices.

Since the original option pricing formula, several writers have extended the application of the theory to other securities. Merton (1974) presents a rigorous derivation of the pricing formula for equity and debt of levered firms, and also prices discount bonds as well as the typical coupon bond. Work by Ingersoll (1977a, 1977b) and by Brennan and Schwartz (1977) applies option pricing techniques to convertible bonds. This research should provide the most-important background toward the eventual extension of option pricing theory to convertible bond valuation. Bartler and Rendleman (1979) discuss the use of option theory to bank loan commitments. Recent articles by Meisner and Labuszewski (1984) and Achour and Brown (1984) have modified the Black-Scholes model

for valuation of options on securities, options of storable commodities, options on futures contracts, "futures-style" options on future contracts, and land options.

Convertible mortgages might also be valued by extending the basic Black-Scholes model. Convertible bonds have been valued by modifying the option pricing formula, and they are quite similar to convertible mortgages. A convertible bond is a straight bond with a warrant attached, while convertible mortgages can be viewed as straight bonds with a long-term European call option(s) attached.

Summary

Although there is little useful literature specifically on convertible mortgage valuation, a review of related literature provides promise for the possible extension of financial theory to convertible mortgage valuation. Discounted cash flow analysis of convertible bonds can be applied to the valuation of the conversion privilege of some mortgages. Option pricing theory also offers a possible answer to this study's primary question of how a convertible bond should be valued.

Notes

1. The market value at the call date, M_m, equals conversion value C_m when g is large enough to make the conversion value exceed the face value prior to expiration. Otherwise, the call price determines market value and m equals the maturity date. The model assumes that the investor does not exercise before the call date, that there is no sinking fund, and that the conversion privilege does not expire prior to maturity.

2. Interest rates are assumed to be constant and known over time. The same interest rate applies to borrowing and lending. No dividends or other payouts are assumed. Costless information, costless transactions, and no taxes are assumed. The derivation of the continuous time formula assumed that the variance of the stock price is both constant and known.

CHAPTER THREE
DISCOUNTED CASH FLOW VALUATION MODEL

Introduction

Discounting cash flows associated with the mortgage (interest payments, participation income, and equity conversion), at an appropriate required rate of return, is one method that can lead to the valuation of convertible mortgages. Both the convertible bond and the convertible mortgage may be viewed as a combination of two more simple securities. These hybrid securities can be evaluated as straight bonds (mortgages) with a call option (land purchase option) attached.

The discounted cash flow model developed in this chapter will allow a lender/investor to price convertible mortgages based upon the lender's required rate of return and his expectation as to future property income and value. The computer program developed allows a great degree of flexibility while requiring limited inputs. The model generates several calculations useful in analyzing a convertible mortgage.

The Model

Discounted cash flow analysis incorporates both the element of risk and timing associated with cash flows. As a

lender, a loan commitment is made today, with the expectation of future cash inflows. The borrower is expected to make interest payments on the loan, perhaps share a portion of the project's income with the lender, and at some future date maybe relinquish ownership of the property itself. With the exception of interest payments there is limited certainty associated with the cash inflows to the lender. Appendix I provides a user-interactive model, written in IFPS (Interactive Financial Planning System), that may be used to calculate the value associated with a conversion option attached to a mortgage.

Required Input Variables

The program requires the user to input the following:

(1) the initial project value,
(2) the loan to value ratio,
(3) the coupon rate of the convertible mortgage,
(4) the required rate of return (interest income),[1]
(5) the required rate of return (participation income),[1]
(6) the required rate of return (conversion value),[1]
(7) the maturity term of the loan,
(8) the date conversion will occur,
(9) the capitalization rate (used to estimate the property's value at conversion),
(10) the annual growth rate of net operating income, and
(11) the rate of participation in net income.

Output Solutions

The model makes ten calculations useful in the analysis of convertible mortgages:

(1) value of convertible mortgage
(2) convertible mortgage/straight mortgage ratio
(3) value of conversion option
(4) cost of conversion option
(5) net option value
(6) option value ratio
(7) lender internal rate of return

(8) implicit annual property appreciation
(9) breakeven conversion value
(10) breakeven required annual property appreciation

A brief explanation of each calculation follows:

(1) Value of Convertible Mortgage--This figure is the present value of interest payments, participation income, and the conversion value (estimated property value times participation rate) discounted at their respective required rates of return.

(2) Convertible Mortgage/Straight Mortgage Ratio--This figure is derived by dividing the value of convertible mortgage (1) by the original loan number.

(3) Value of Conversion Option--The figure generated by this calculation compares the value of convertible mortgage (1) to the present value of the convertible mortgage as if it were a straight mortgage paid off at conversion (not converted).

(4) Cost of Option--Since convertible mortgages are originated at discounted interest rates, the cost of the privilege to convert is equal to the present value of lost interest income minus the present value of participation income.

(5) Net Option Value--This calculation is the value of conversion option (3) minus the cost of the option (4).

(6) Option Value Ratio--This ratio is determined by dividing the value of conversion option (3) by the cost of the option (4).

(7) Lender Internal Rate of Return--The IRR calculation represents the average annual return on the convertible mortgage using forecast cash inflows.

(8) Implicit Annual Property Appreciation--This is an IRR calculation to determine the implicitly assumed property appreciation. The original loan amount is the cost figure while the projected property conversion value equals the cash flow.

(9) Breakeven Conversion Value--This figure represents the property conversion value that would be required to equate the convertible mortgage to a straight mortgage yielding the required rate of return. This calculation is made by adding the original loan amount to the compounded future value at conversion of the cost of option (4).

(10) Breakeven Required Annual Property Appreciation--This calculation is the same as that done in (8) except breakeven conversion value (9) is considered the cash inflow.

Model Extension

In the standard model a growth rate is specified in order to predict future net operating incomes, and therefore estimated property conversion value, since that value is the net operating income in the conversion year capitalized at ten percent. A more sophisticated approach to forecasting future cash flows is also available in the model. Specifically, Monte Carlo simulations are appropriate when

it is either impossible or inadequate to assign single-point estimates to a variable such as income growth. Thus, when available data reflect a high degree of uncertainty or when a number of alternatives exist, major decisions and new situations often benefit from a Monte Carlo simulation.

The Monte Carlo simulation procedure can solve the model a number of times based on different random values generated within the distribution. The Monte Carlo process then summarizes the results from all these model solutions, allowing one to make probability statements about the analysis.

Application of the model: A Case Example

Input/Output

To illustrate the operation of the model, a convertible mortgage has been valued for the particular set of parameter values given in Table 1. It is believed that this example is representative of the typical convertible mortgage being originated today.

Table 1. Parameters of case example.

Variable	Value
Initial Project Value	$10,000,000
Loan to Value Ratio	.80
Coupon Rate	.105
Required Rate of Return (interest income)[1]	.135
Required Rate of Return (participation income)[1]	.15
Required Rate of Return (conversion income)[1]	.15
Loan Maturity Date	20 years
Conversion Date	5 years
Capitalization Rate[2]	.10
Growth Rate of NOI	.05
Participation Rate	.80

The example represents an $8,000,000 loan discounted 300 basis points below the required market rate of return for interest income. It is assumed that 15 percent is an adequate discount factor (required rate of return) on forecasted participation and conversion value cash flows. The loan matures in twenty years; however, the first option to convert to an equity position occurs in year five. The project's annual net operating income is projected to grow at a five percent rate. The conversion value of the property will equal the fifth year's net operating income capitalized at a 10 percent rate. Table 2 depicts the model's value solutions for the case example.

Table 2. Convertible mortgage valuation.

Output	Value
Value of Convertible Mortgage	$7,834.778
Convertible Mortgage/Straight Mortgage Ratio	.979
Value of Conversion Option	$ 489,730
Cost of Conversion Option	$ 385,088
Net Option Value	$ 104,642
Option Value Ratio	1.272
Lender IRR	.1404
Implicit Annual Property Appreciation	.0235
Breakeven Conversion Value	$8,667,973
Breakeven Required Annual Property Appreciation	.0162

Interpretation of Output

Data from Table 2 suggest that the convertible mortgage being analyzed is not an attractive investment. The present value of this convertible mortgage is $7,834,778. This does not compare favorably with the loan amount of $8,000,000.

If the forecast cash flows occur, the lender's internal rate of return (IRR) will be 14.04 percent. Since multiple discount rates are used in the model, it is difficult to evaluate the project based on lender IRR. Obviously the 14.04 percent rate of return of the overall project is unacceptable, however, since the convertible mortgage/straight mortgage ratio is only .979. A lender would be better advised to make the typical mortgage loan than to originate the convertible mortgage as packaged in this case example.

Analyses of those outputs which specifically value the conversion option feature show an interesting situation. The right to switch from a debt position to one of ownership is worth $489,730. The cost of the conversion option is only $385,088, which produces a net option value of $104,642 (489,730 - 385,088). An option ratio of 1.272 (489,730/385,088) appears to be favorable.

Why are the outputs associated with the conversion feature positive when lender IRR and the convertible mortgage/straight mortgage ratio are not favorable? The answer lies in an analysis of the various cash flows forecast for this convertible instrument. The predicted property appreciation and increasing participation income are quite favorable, but the 300 basis point discount on the loan coupon rate represents a considerable amount of lost income. By discounting the loan three percentage points, $240,000 of interest income is lost annually. Participation income never quite compensates for this loss, and since the

conversion value cash flow is five years in the future, its value is diminished significantly. This is particularly true since these two cash flows are discounted at a 15 percent required rate of return. Although the privilege of conversion is favorable, lost interest income in conjunction with higher risk-adjusted discount rates makes the total loan unacceptable.

The last group of outputs calculated by the model relate to the property's appreciation. The forecast conversion value ($8,985,022) implies that the property will appreciate at an annual rate of 2.35 percent. For the investment to breakeven, a conversion value of $8,667,973 is required in year five. To breakeven the property must annually grow in value by 1.62 percent.

The output generated by the model indicates that this particular convertible mortgage decreases the expected yield to a lender. The yield is not greatly decreased, however, which suggests that certain modifications to the loan contract might be desirable, for example a reduction in the coupon rate discount of an increase in the lender's rate of participation in operational cash flows. Ultimately a mix of features should be possible that would be advantageous to both the lender and the borrower. By carefully structuring the loan agreement, yields can be increased, or at least made acceptable, for both parties at interest.

Sensitivity Analysis

In addition to providing an estimate of the value of a particular convertible mortgage, the model may be used to assess the sensitivity of the lender's IRR to changes in environmental, project, and mortgage parameters. The results of some representative calculations are shown in Table 3. In the table various parameters are changed one at a time.

The first five parameters shown are specifically related to the terms of the loan. These variables would be subject to negotiation before the mortgage agreement was signed. The last parameter subjected to sensitivity analysis, growth rate of NOI, is exogenous to the terms of the loan. It is perhaps the most important determinant in developing the proper mix of loan terms, since higher growth expectations allow the lender to make greater concessions in the areas of coupon rates, loan to value ratios, and participation requirements.

The model appears to be particularly sensitive to change in two variables: loan to value ratio and growth rate of NOI. The IRR associated with a loan to value ratio of 70 percent is 11.3 percent greater than the IRR at a 90 percent loan to value ratio. In fact, at a 90 percent loan to value ratio the loan is not acceptable, whereas the lower ratio yields a return that is above the required rates of return on all cash flows. This difference indicates the importance of participation income and the fact that less

Table 3. Sensitivity analysis.

Variable	Value of Conv. Mortgage	CM/Str. Mtg. Ratio	Value of Conv. Option	Cost of Option	Net Option Value	Option Value Ratio	Lender IRR	Implicit Annual Property Appreciation	Breakeven Conv. Value	Breakeven Required Annual Property Appreciation
					OUTPUT					
Conversion Date										
5th year (Std.)	$7,834,778	.9793	$489,730	$385,088	$104,642	1.272	.1404	.0235	$8,667,973	.0162
7th year	8,067,885	1.008	716,531	359,142	357,389	1.995	.1474	.0310	8,711,216	.0122
10th year	8,346,054	1.043	857,093	233,596	623,497	3.669	.1520	.0367	8,309,540	.0038
Loan to Value Ratio										
.70	7,193,118	1.028	428,514	(736)*	429,249	(582.5)*	.1542	.0235	6,905,267	(.0027)*
.80 (Std.)	7,834,778	.9793	489,730	385,088	104,642	1.272	.1404	.0235	8,667,973	.0162
.90	7,836,564	.9796	489,730	191,148	298,582	2.562	.1385	.0235	8,205,591	.0051
Coupon Rate/ Required Rate (int. inc.) (constant 300 basic pt. spread)										
.095/.125	7,836,564	.9796	489,730	191,148	298,582	2.562	.1385	.0235	8,205,591	.0051
.105/.135 (Std.)	7,834,778	.9793	489,730	385,088	104,642	1.272	.1404	.0235	8,667,973	.0162
.115/.145	7,822,304	.9778	489,730	579,821	(90,091)*	.8446	.1423	.0235	9,130,938	.0268
Coupon Rate/ Reg. Rate (int. inc.) (spread differential)										
.10/.135	7,803,058	.9754	489,730	416,808	72,922	1.175	.1395	.0235	8,714,013	.0172
.105/.135 (Std.)	7,834,778	.9793	489,730	385,088	104,642	1.272	.1404	.0235	8,667,973	.0162
.11/.135	7,866,499	.9833	489,730	353,367	136,363	1.386	.1414	.0235	8,621,934	.0151

Table 3 (Cont'd.)

Variable	Value of Conv. Mortgage	CM/Str. Mtg. Ratio	Value of Conv. Option	Cost of Option	Net Option Value	Option Value Ratio	Lender IRR	Implicit Annual Property Appreciation	Breakeven Conv. Value	Breakeven Required Annual Property Appreciation
					OUTPUT					
Required Rates (int. inc./part. inc & EV)										
.135/.14	$8,047,330	1.006	$511,590	$371,931	$139,659	1.375	.1404	.0235	$8,681,216	.0165
.135/.15 (Std.)	7,834,778	.9793	489,730	385,088	104,642	1.272	.1404	.0235	8,667,973	.0162
.135/.16	7,632,908	.9541	468,982	397,700	71,282	1.179	.1404	.0235	8,654,523	.0159
Growth Rate of NOI										
.025	7,309,934	.9137	79,242	499,445	(420,202)*	.1587	.1207	.0040	8,897,986	.0215
.05 (Std.)	7,834,778	.9793	489,730	385,088	104,642	1.272	.1404	.0235	8,667,973	.0162
.075	8,395,326	1.049	930,609	265,420	665,190	3.506	.1601	.0429	8,427,279	.0105
Participation Rate										
.60	7,722,566	.9653	489,730	497,300	(7,570)*	.9848	.1364	.0235	8,893,673	.0214
.70 (Std.)	7,834,778	.9793	489,730	385,088	104,642	1.272	.1404	.0235	8,667,973	.0162
.80	7,778,672	.9723	489,730	441,194	48,536	1.110	.1384	.0235	8,780,823	.0188

*( ) results because the cost of option is negative; i.e., there is no loss income.

interest income is lost when lower loan to value ratios exist. The 15.42 percent indicated IRR may be misleading, however, since the standard model incorporates an 80 percent participation rate.

As expected, the parameter that impacts upon the lender's IRR most significantly is the projected growth rate of NOI. This variable can cause cash flows to change drastically. Since participation income is a function of NOI, the faster the forecast rate increases, the greater the cash flow. Conversion value is also directly related to NOI growth. Recall that the estimated NOI on the date of conversion is capitalized at 10 percent to determine the estimated value of the property. The standard growth rate is 5 percent. If the growth rate is 2.5 percent, the project is not profitable (12.07 percent IRR). When NOI is forecasted to grow at an annualized 7.5 percent rate, however, the loan's IRR is 16.01 percent. This 4 percentage point difference represents a 32.6 percent difference in yield.

Monte Carlo Simulation

In general, a project's risk depends on both its sensitivity to changes in key variables and the range of likely values of these variables, i.e., the variables' probability distributions. Monte Carlo simulation overcomes the inadequacies of single-point estimates which often amount only to safety margins, worst-case extremes, or

intuitive assessments. The first step in a Monte Carlo simulation is to specify a probability distribution for each of the key variables in the analysis. The simulation procedure solves the model a number of times based on different random values generated within the distribution. The Monte Carlo process summarizes the results from all these solutions, allowing one to make probability statements about the analysis.

A demonstration of a Monte Carlo simulation is presented in Table 4. Since changes in the growth rate of NOI cause considerable volatility in lender IRR this variable is examined in the simulation. Required inputs are the mean and standard deviation for this variable. For illustrative purposes, assume a growth rate of NOI of 5 percent and a standard deviation of .025 percent which represents a 50 percent deviation from the mean. These figures imply an expectation of a 5 percent growth in NOI, but allow the rate to range from 2.50 to 7.50 percent within a 68 percent confidence interval. This distribution range allows the investor to view the probability of each outcome's occurrence when there is a chance that the growth rate will be as much as 50 percent higher or lower than the 5 percent expected rate. Though a normal distribution is assumed for illustrative purposes, the model allows several other distribution patterns.

Table 4 shows the probability of each output value being greater than the figure indicated. For example, there is a 90 percent probability that the value of the convertible mortgage will be greater than $7,435,000, but only approximately a 25 percent probability that it will exceed $8,000,000 (the original loan amount). Obviously this type of output can provide investors with an important insight into the risk associated with convertible mortgages. The sample statistics portion of the table provides additional insight into the riskiness of this particular convertible mortgage.

Summary

While retaining many of the characteristics of straight mortgages, the convertible mortgage offers in addition upside potential associated with the underlying property. The convertible mortgage valuation model developed in this chapter values the mortgage and provides other pertinent information for decision-making purposes. The model which focuses on discounted cash flow analysis of mortgages, is flexible enough to handle a wide range of alternative lender/borrower option terms. The input required to generate output is quite minimal. The output from the model should facilitate better lending decisions.

The model not only makes specific calculations of ten outputs, it also allows the user to incorporate risk analysis into the process. Sensitivity analysis

Table 4. Monte Carlo simulation.

Frequency Table

Probability of value being greater than indicated

	90	80	70	60	50	40	30	20	10	
Conversion value										
5	8328	8602	8772	8916	9011	9085	9223	9332	9544	*1000
Value of convertible mortgage										
5	7435	7605	7717	7776	7856	7892	7973	6061	8214	*1000
Cost of option										
5	279737	325757	357357	369156	383877	396695	410562	429347	468575	
Lender IRR										
5	.126	.132	.136	.138	.141	.143	.145	.149	.154	
Forecasted annual property appreciation										
5	.008	.015	.019	.022	.024	.026	.029	.031	.036	
Breakeven conversion value										
5	8456	8549	8612	8636	8666	8691	8719	8757	8836	*1000
Annual property appreciation required to breakeven										
5	.011	.013	.015	.015	.016	0.17	.017	.018	.020	
CM/straight mort. ratio										
5	.929	.951	.965	.972	.982	.987	.997	1.008	1.027	
Option value ratio										
5	.359	.708	.969	1.120	1.322	1.446	1.685	1.994	2.836	
Net option value										
5	295496	124941	12705	45713	125972	162232	242422	330720	483746	

Sample Statistics

	Mean	Std Dev	Skewness	Kurtosis	10PC Conf Mean	90PC
Conversion value						
5	8982934	451655	.0	3.0	8925122	9040746
Value of convertible mortgage						
5	7840690	288177	.0	2.9	7803804	7877577
Cost of option						
5	378137	69599	-.2	2.7	369229	387046
Lender IRR						
5	.1405	.0105	-.1	2.9	.1391	.1418
Forecasted annual property appreciation						
5	.0232	.0103	-.1	3.0	.0219	.0246
Breakeven conversion value						
5	8653994	139989	-.2	2.7	8636075	3671913
Annual property appreciation required to breakeven						
5	.0158	.0033	-.3	2.8	.0154	.0162
CM/straight mort. ratio						
5	.9801	.0360	.0	2.9	.9755	.9847
Option value ratio						
5	1.452	.9514	1.2	5.0	1.221	1.574
Net option value						
5	110554	288177	.0	2.9	72668	147441

capabilities provide a means of quickly determining the impact of changes to any given variable. Monte Carlo simulation provides even greater risk analysis. By incorporating the simulation technique into the calculation of outputs the impact of uncertainty can realistically be assessed.

Testing the model against convertible mortgages that have actually been originated appears to be the logical next step. To this end, a questionnaire was developed which gathered convertible mortgage data from a wide range of large institutional investors. Based on the data provided from the questionnaire, an attempt was made to determine how lenders price convertible mortgages. This analysis is developed in chapter six.

Another extension of this research relates to option pricing theory. Models for the pricing of options have proliferated in recent years. A convertible mortgage can be viewed as a straight mortgage with an option attached. Chapter four demonstrates the application of option pricing theory to the valuation of convertible mortgages.

Notes

1. Required Rates of Return: Obviously the risks associated with various cash flows of a real estate project differ. Because each cash flow (interest income, participation income,and conversion value) is independent, although perhaps closely related, the model allows for different discount rates. The case example discounts the interest income at 13.5 percent. This required rate represents what is believed to be a fair approximation of current market rates on straight mortgages of similar risk and maturity to the convertible mortgage being valued. A 15 percent required rate of return was chosen to discount both participation and conversion value cash flows. The same rate was used for both flows since they are so closely related. Conversion value is a function of income. In fact, conversion value is calculated by dividing forecasted net operating income by a capitalization rate. The 15 percent rate represents a close approximation of what actual returns have been in commercial real estate over the long term (Hoag; Miles and Esty). An investor using the model will choose the rate(s) he deems appropriate.

2. The capitalization rate is the percentage of return required from an investment which provides for investor income, principal and interest of debts, and loss in value from depreciation.

CHAPTER FOUR
APPLICATION OF OPTION PRICING THEORY TO CONVERTIBLE MORTGAGE VALUATION

Introduction

Convertible mortgages are unique hybrid securities. Their uniqueness stems from the fact that, unlike most mortgages, the mortgagee has a right to change his position from creditor to owner of the property which collateralizes the loan. The right to convert to an ownership interest in the property represents an option. It is this portion of the convertible mortgage which makes valuation difficult. Fortunately, during the past two decades considerable academic research on options has been conducted.

Almost as if it were timed to coincide with the opening of the Chicago Board Options Exchange, a theoretical valuation formula for stock options, derived by Fisher Black and Myron Scholes, was published in The Journal of Political Economy for May-June 1973. Growing investor interest in options has been paralleled by research breakthroughs on the nature of option pricing. Since an option can be formed on any underlying security many financial instruments can be considered as options. Thus, option theory is of great

interest in the academic financial community because it can be applied to a wide range of financial instruments.

This chapter explores the application of option pricing theory to valuation of convertible mortgages. First, the Black-Scholes option pricing formula will be described. The Black-Scholes model provides the basic explanation of how option theory works. From this foundation option pricing theory is extended to value convertible mortgages. A convertible mortgage valuation formula is developed in three stages to separately value the mortgage cash flows (interest income, participation income, and conversion value). After a theoretical valuation model is developed, the practical application of option pricing theory to convertible mortgages is discussed. The underlying assumptions of option theory are viewed in light of the realities of convertible mortgages. Finally, a representative convertible mortgage is valued using the model developed. Sensitivity analysis also accompanies this test.

The Black-Scholes Option Pricing Model

It is important to understand intuitively what the Black and Scholes model implies and what it assumes since it will be the basis for the valuation approach used in the research. Black and Scholes (1973) developed a model of what the price of a call option should be in equilibrium under certain assumptions. The assumptions include the following:

1. Short sales are unrestricted.

2. There are no taxes or transaction costs.
3. Trading is continuous.
4. There is a constant risk-free rate.
5. The stock price is continuous.
6. The stock pays no dividends.
7. The option can only be exercised at maturity (European type).

In addition, the solution was derived assuming that future stock prices will follow a log-normal process; however, extensions have relaxed that assumption and considered other processes (Cox and Ross, 1976).

The contribution of Black and Scholes was not in the uniqueness of these assumptions. Previous option valuation models such as those of Sprekle (1964), Boness (1964) and Samuelson (1966) used similar assumptions. The insight of Black and Scholes (1973) was based on the observation that the prices of a security and a call option on the security move in the same direction. Therefore, positions long in one and short in the other may be formed to create hedges which are riskless for small changes in stock price. For example, one could hold the stock long and write call options on the stock. Then an increase in the stock price would increase the value of the long side of the portfolio. But the stock price increase would make the value of the call option greater to its holder and, therefore, less to its writer

that the short side of the portfolio (the call written) would decrease in value.

There would exist some ratio of stock held to call options written for which the offsetting changes in value would be exactly the same, and the portfolio would be riskless. Therefore, with only the basic economic assumption that identical returns be valued identically, the return to the hedged portfolio must be the riskless rate. By using this result and the previously mentioned assumption concerning the process governing stock prices, Black and Scholes are able to express the equilibrium value of a call option as a function of only the following variables:

1. the stock price,
2. the instantaneous variance of percentage returns on the stock price,
3. the exercise price of the option,
4. the time to maturity of the option,
5. the risk-free interest rate.

An important point for tests and applications of the model is that all of the variables except the variance are directly observable and the variance may be estimated based on historical price sequences. In addition, no assumptions need be made concerning investor expectations or risk preferences.

An outline of the development of the options pricing model is presented below. This presentation follows an

outline of continuous-time models written by Stephen Figlewski (Figlewski, 1977). The following notation will be used.

C = value of call option
S = price of stock
T_2 = time to expiration of option
σ = variance of the rate of return on S
X = option exercise price
r = risk-free interest rate

Assume that the return on a stock follows a diffusion process which is log-normal.

$$\frac{S_{t+dt} - S_t}{S_t} = \frac{dS}{S} = \mu dt + \sigma dz \qquad (4.1)$$

This says that the percentage change in the value of the stock from time t to time t+dt consists of a nonstochastic return over time, μdt, and a random term σdz where dz is the standard Weiner process. The term dt refers to an infinitesimal change in the time index.

dz is n $[0,\sqrt{dt}]$

σdz is N $[0,\sigma^2\sqrt{dt}]$

so $\frac{dS}{S}$ is N $[\mu dt,\sigma^2\sqrt{dt}]$

The hedged portfolio suggested by Black and Scholes can be expressed as:

$$V_H = S\,Q_S + CQ_C \qquad (4.2)$$

V_H = value of hedged position

S = stock price

C = call option price

Q_S and Q_C = quantity of stock and options respectively in portfolio.

Taking the derivative of (4.2) gives

$$dV_H = dSQ_S + dCQ_C. \tag{4.3}$$

For the hedge to be perfect, the change in the hedge value V_H must be equal to zero. Set (4.3) equal to zero and solve for the quantities of stock and options necessary for a perfect hedge.

$$dSQ_S + dCQ_C = 0$$

$$Q_S = -Q \frac{\mu C}{\mu S}.$$

Arbitrarily normalize one quantity.

Let $Q_C = 1$, then $Q_S = -\frac{\mu C}{\mu S}$.

Using Ito's Lemma from stochastic calculus to solve (4.3) for dC gives

$$dC = \frac{\mu C}{\mu S} dS + \frac{\mu C}{\mu t} dt + 1/2\ S^2\sigma^2 \frac{\mu^2 C}{\mu_S{}^2} dt. \tag{4.4}$$

Substituting into (4.3) with the hedge ratio calculated above, then,

$$dV_H = \left(\frac{\mu C}{\mu t} + 1/2\ S\sigma^2\sigma \frac{\mu^2 C}{\mu S^2}\right)dt. \tag{4.5}$$

Dividing (4.5) by V gives the rate of return on the portfolio which, since the portfolio is riskless given the proper weights of stock (Q_S) and calls (Q_C), must be riskless rate, rdt.

$$\frac{dV_H}{V} = \frac{\left(\frac{\partial C}{\partial t} + 1/2\ S^2 s^2\ \frac{\partial^2 C}{\partial S^2}\right)dt}{Q_S\ S + Q_C\ C} = rdt \qquad (4.6)$$

Since $Q_S = -\frac{\partial C}{\partial S}$ and $Q_C = 1$

$$\frac{\left(-\frac{\partial C}{\partial t} + 1/2\ \sigma^2\ S^2\ \frac{\partial^2 C}{\partial S^2}\right)dt}{-\frac{\partial C}{\partial S}\ (S) + C} = rdt. \qquad (4.7)$$

Cancelling dt and rearranging,

$$\frac{\partial C}{\partial t} = rC - rS\ \frac{\partial C}{\partial S} - 1/2\ S^2\sigma^2\ \frac{\partial^2 C}{\partial S^2} \qquad (4.8)$$

which is the fundamental partial differential equation of option pricing.

At maturity (*) the option (C*) will be valued as follows:

$$C^* = \begin{cases} S^* - X & \text{if } S^* > X \\ 0 & \text{if } S^* < X. \end{cases} \qquad (4.9)$$

The value of the option, C, at any time may be found by solving (4.8) subject to the maturity constraint (4.9). Black and Scholes use a change of variables to transform (4.8) into the heat exchange equation for which a solution is known. The solution reached is

$$C = S \cdot N\left(\frac{\ln \frac{S}{X} + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}\right) - e^{-rt} X \cdot$$

$$N\left(\frac{\ln \frac{S}{X} + \left(r - \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}\right) . \qquad (4.10)$$

where N [·] denotes the cumulative standard normal distribution.

A more intuitive approach to the solution was found by Cox and Ross (1976). They reasoned that since nothing in the hedge equation (4.2) depends on the risk preferences of investors, one can assume any attitude toward risk which makes a solution easier and then generalize with no difficulty. They assumed risk neutrality. Risk neutrality implies all assets yield the same return, namely the riskless rate, r.

$$\frac{dS}{S} = r dt + \sigma dz \qquad (4.11)$$

(note, this is (4.1) with r replacing μ).

The boundaries at maturity are known (4.9), and the process governing returns on the stock is assumed

to be log-normal. Therefore, the expected value of the call at maturity can be expressed as

$$E[C^*] = \int_X^\infty (S^* - X)\, g(S^*)\, dS^* \qquad (4.12)$$

where * denotes maturity and $g(S^*)$ is the density of the log-normal process assumed to be followed by the stock price. The solution for the value of C at any time before maturity requires that (12) be solved and discounted back in time. Since S^* is log-normally distributed, $\ln S^*$ is normally distributed and with a change in variables the function $g(S^*)$ can be found. Some additional changes in variable and standardizations are required to complete the integration of (4.12), but the solution reached is identical to that of Black and Scholes given in (4.10).

To get an intuitive feel for the meaning of (4.10), assume risk neutrality. Then the first term is the discounted expected value of the stock price at maturity, given that the stock price exceeds the exercise price, times the probability that the stock price will be greater than the exercise price. The second term is the discounted expected value of the exercise price times the probability (the $N[\cdot]$ element) that the stock price will exceed the exercise price. Note that this interpretation holds only under risk neutrality (Smith, 1976). Static comparisons reveal that the value of the call option is increased by increases in the risk-free rate (r), the variance (σ^2), the

stock price (S) and the time to maturity (T). The call's value is decreased by increases in the exercise price (X).

Extension of Black-Scholes To Convertible Mortgages--A Model

In addition to deriving the options pricing model Black and Scholes pointed out that the approach could be used to find equilibrium values for other complex contingent claims assets. Convertible mortgages meet the primary condition in that their values depend on the values of a more basic asset. However, the convertible is considerably more complex than the simple option; therefore, applying the model is not straightforward.

To investigate the possibility of utilizing option pricing theory to convertible mortgage valuation one must first derive a payoff function--a function relating the payoff to the contingent claim holder (the lender) as a function of the value of the underlying asset (the property). Convertible mortgages have a number of peculiar characteristics which prohibit usage of simple option pricing techniques. They are hybrid instruments in the sense that they contain equity and debt aspects. The mortgagee has, apart from a regular lending position, also a potential equity stake in the proceeds of the underlying asset.

Holders of convertible mortgages might typically expect three distinct cash flows from their investment.

mortgage will generate an interest payment, possibly participation income, and potentially the appreciated value of the property upon conversion. Each of these elements can be priced separately. This separation theorem relies on the findings of Modigliani and Miller (1959) that the value of the firm or asset, is independent on the way in which it is financed. The same arguments hold here; the property value is independent of the way in which it is financed. It matters not whether this be debt and equity or total equity if conversion occurs.

Since publication of the Black-Scholes (1973) paper in which the pricing models for simple put and call options were originally derived, there has been much work employing the continuous time, option pricing analysis which they developed. Various researchers have successfully extended the basic option pricing model to value other financial claims. The model which follows builds upon previous work by identifying the elements of a convertible mortgage and applying appropriate pricing techniques already known for each element.

A Basic Model

Elements of a Convertible Mortgage

As previously mentioned, a convertible mortgage has the potential for three distinct cash flows: interest income, participation income, and property value appreciation if conversion occurs. The first cash flow, interest payments,

relates to a standard collateralized loan contract and can be valued as if it were corporate debt (bonds). Like corporate bonds, convertible mortgages are typically interest-only loans with the principal repaid at maturity. Participation income is often calculated as a constant percentage of the total dollars of revenue generated by an investment property (gross revenue). If it is assumed that value is directly related to revenue, participation income can be derived as a function of property value. Participation income is thus similar to a dividend yield. The conversion privilege will be valued as a European call option which allows the lender to assume an equity stake in the property if the option is exercised.

The typical convertible mortgage has the following elements:

1. a standard collateralized loan
2. participation yield
3. an option to convert the loan into an ownership stake in the asset.

Let's call 1. D for debt; 2. P for participation income; and 3. C for the call option. Fortunately, pricing techniques are known for each of these elements.

The convertible mortgage is clearly a combination of debt, a participation cash flow, and an option on the equity of the underlying property. The price of the convertible will thus reflect all of these features. Since both the

debt and the option features of the convertible can be priced using the Black-Scholes approach, one would expect that the convertible mortgage, as a combination of these, can also be priced. Indeed it can, and the next section of this chapter prices each of the mortgage elements.

Collateralized Loan (Debt)

Black and Scholes (1973) suggest that the option pricing model can be used to price both the debt and equity of a levered firm. Certain restrictive assumptions apply: (1) The firm issues pure discount bonds. The bonds mature at t*, T time periods from now, at which time the bondholders are paid (if possible), and the residual is paid to the stockholders. (2) The total value of the firm is unaffected by capital structure (to apply stochastic calculus it must be assumed that the process describing the total value of the firm can be fully specified without reference to the value of the contingent claims). (3) Homogeneous expectations exist about the dynamic behavior of the value of the firm's assets (the distribution at the end of any finite time interval is log-normal with a constant variable rate of return). (4) There is a known constant riskless rate, r. (5) The dynamic behavior of the value of the assets is independent of the value of the probability of bankruptcy. (6) There are no costs to voluntary liquidation or bankruptcy. Bankruptcy is defined as the state in which the borrower's

assets are less than the promised repayment amount of a maturing loan.

Issuing bonds is equivalent to the stockholders' (borrowers) selling the assets of the firm to the bondholder (lender) for the proceeds of the issue plus a call option to repurchase the assets of the firm from the bondholders with an exercise price equal to the face value of the bonds.

The Black-Scholes call pricing model thus provides the correct valuation of the equity. Applying the Black-Scholes call option solution yields

$$E = VN\left(\frac{\ln (V/X) + (r+\sigma^2/2)\ T}{\sigma\sqrt{T}}\right)$$

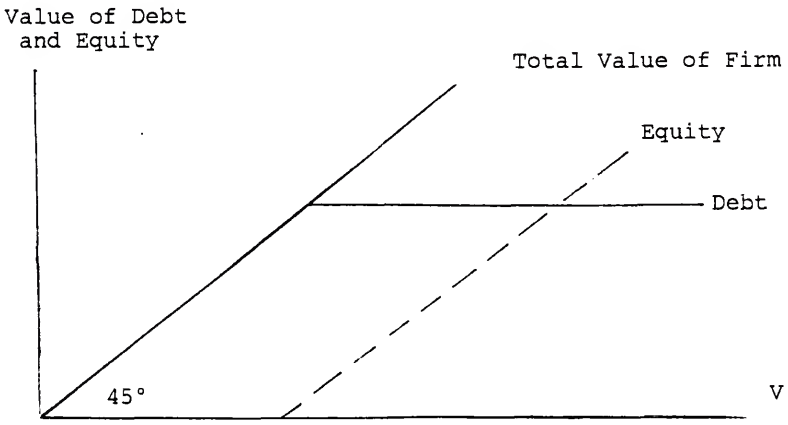
$$-e^{-rt}XN\left(\frac{\ln (V/X) + (r-\sigma^2/2)\ T}{\sigma\sqrt{T}}\right), \qquad (4.13)$$

where E is the value of the equity of the firm, V is the value of the assets of the firm, X is the face value of the debt of the firm, σ^2 is the variance rate on V, and T is the maturity date of the debt. The same relationship exists when a real estate developer/investor borrows funds which are collateralized by the property.

As would be expected from the previous discussion, this formula is the same as the call option formula (4.10) derived earlier in this chapter, but with the value of the firm, V, replacing the price of the stock, and with the debt payment X being used as the exercise price. As X goes to

zero, so that the firm approaches an unlevered financing position, the value of the equity equals the total value of the firm.

Our interest is in pricing the loan, and similar analysis can be done for the debt of the firm since the debt is complementary to the equity. What is gained by one is lost to the other. The terminal condition for the debt of the firm is X = Min (V,X). Therefore, the debtholder gets either the amount due on the bond, X, or the value of the firm, V, which ever is less. This condition is illustrated below.



Because this condition differs from the terminal condition of the call option, the pricing of the debt will differ from that of the equity. The debt is a contingent claim, its final value being contingent on the end-of-period value

of the firm. Therefore the Black-Scholes methodology can be employed to price it.

Merton (1974) derived the formula for the value of debt, D, as D = V - E

$$= VN\left(\frac{-\ln (V/X) - (r+\sigma^2/2)\ T}{\sigma\sqrt{T}}\right)$$

$$+ e^{-rt}XN\left(\frac{\ln (V/X) + (r-\sigma^2/2)\ T}{\sigma\sqrt{T}}\right). \qquad (4.14)$$

Equation (4.14) is the correct method for calculating the value of debt if the original Black-Scholes assumptions are met. Remember, however, that one of those assumptions was that the firm pays no dividends or any other payout. The participation income received by the lender in a typical convertible mortgage violates the basic Black-Scholes restriction of no payouts. An adjustment for this type of situation was made by Merton (1973). Merton relaxed the no payout assumption to deal specifically with dividends. His extension of the basic Black-Scholes model can also be used to reflect the effect of participation income cash flows on the value of debt and the conversion option. Funds paid out by a firm (property), be they dividends or participation fees reduce the value of the firm's remaining assets. This action increases the probability of loan default.

Merton's solution was to develop a rather specific dividend policy--dividends are paid continuously such that

the dividend yield is a constant function of the stock price. This suggests that there is a constant dividend yield (δ). This rate, like an interest rate, represents the period earnings from dividends. Participation income can also be considered to yield a constant percent on the underlying property value. This notion is discussed in detail when participation income is valued.

To modify the Black-Scholes formula to include this rate, we replace V in the first term of the formula with $Ve^{-\delta T}$, and replace the interest rate r with $r - \delta$ in both parts of the formula. Essentially these modifications reduce the firm value V by the amount of the payout, and adjust the return from the riskless hedging strategy accordingly.

The modified debt formula becomes

$$D = Ve^{-\delta T}N\left(\frac{-\ln (V/X) - (r-\delta+\sigma^2/2)\ T}{\sigma\sqrt{T}}\right)$$

$$+ e^{-rt}XN\left(\frac{\ln (V/X) + (r-\delta-\sigma^2/2)\ T}{\sigma\sqrt{T}}\right) \qquad (4.15)$$

$$= D(V,X,T,\sigma^2,r,\delta)$$

Effects of changes in parameters on the value of the loan can be studied by calculating partial derivatives of D:

$$\frac{\partial D}{\partial V},\ \frac{\partial D}{\partial X} > 0 \text{ and } \frac{\partial D}{\partial T},\ \frac{\partial D}{\partial \sigma^2},\ \frac{\partial D}{\partial r},\ \frac{\partial D}{\partial \delta},\ < 0$$

These partial effects suggest that an increase in V directly increases the value of E and increases the debt coverage ratio, thereby lowering the probability of default and increasing the value of D. An increase in the promised repayment amount X increases the value of D and naturally decreases the value of E. Longer dates to maturity and increases in the riskless rate lower the present value of D and increase the value of E. An increase in the time to maturity or in the variance rate increases the dispersion of possible values of V at maturity of the debt. Since debtholders have a maximum payment which they can receive, X, an increase in the dispersion of possible outcomes increases the probability that $V < X$, thereby increasing the probability of default which lowers the value of D and increases the value of E. If δ increases, the expected price appreciation of the property falls. Thus the expected value of the loan at the expiration of the loan will be less, and default is more likely. Tnis decreases the value of the debt.

Participation Income

Participation income can be calculated several different ways. In the discounted cash flow model developed in the previous chapter the participation was determined by subtracting debt payments from net operating income and then multiplying this sum by the loan to value ratio. Gross income is also frequently used as the basis for calculating

participation income. Gross income is relatively easy to quantify and, therefore, is a convenient basis. Usually the cash flow represents a fixed percentage of the entire gross income from a property; perhaps 2 or 3 percent.

The adjustment that Merton made to the basic Black-Scholes model related to dividend payouts. Remember that δ was a payout yield calculated by dividing the payout by the value of the asset. Merton's adjustment is needed to reflect the payout's effect on asset value. Regrettably δ relates payout to value which suggests that the model being developed should relate participation income to property value and not gross income. This is not really a great problem, however, as it is not difficult to equate a fixed percentage of gross income to a constant return on property value. Value is a function of gross income, and it can be assumed that the relationship is relatively constant over time. In this model it will be assumed that the relationship between property value, gross revenue, and participation income is constant. This allows participation income to be stated as a fixed percentage of value.

Participation income is the easiest element of the convertible mortgage to price. Using simple discounted cash flow techniques, P can be expressed as:

$$P = \alpha V(1-e^{-\delta T})$$

where α is the lender's portion of gross revenues (usually α equals the original loan to value ratio), and δ is participation income divided by property value.

In general form the equation for participation income can be expressed as

$$P(\alpha V, T, \sigma^2),$$

where $\frac{\partial P}{\partial \alpha V}, \frac{\partial P}{\partial \sigma^2}, \frac{\partial P}{\partial \delta} > 0$; and $\frac{\partial P}{\partial T} < 0$.

Option to Convert Loan

The final, most interesting and important element of a convertible mortgage is the conversion privilege given the lender. This option allows the lender to surrender his debt position and acquire an equity position in the collateral asset at a date T, where T is smaller than the maturity date of the loan, t*.

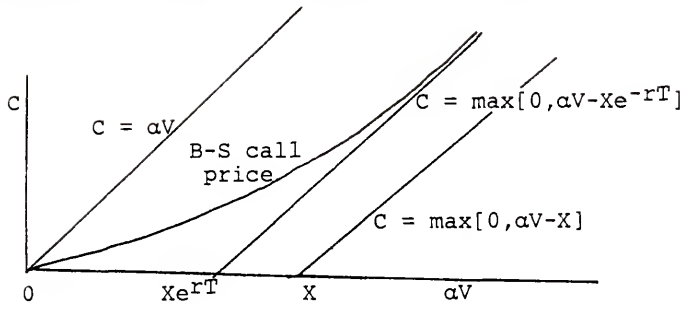
The convertible mortgage is equal in value to a portfolio containing a discount bond as described in the previous section and a European call option entitling the owner to purchase a fraction, α, of the equity of the collateralized property upon payment of the exercise price equal to the principal X of the debt. A convertible mortgage will thus be priced at a premium above a simple discount bond, with the premium being worth the value of the conversion feature.

The conversion privilege (a European call option) was derived earlier in the chapter as formula (4.10). The Black-Scholes solution to the European call pricing problem for convertible mortgages requires only the minor adjustments of changing the stock price(s) to αV which represents the conversion value in the property. The new equation would take the following form:

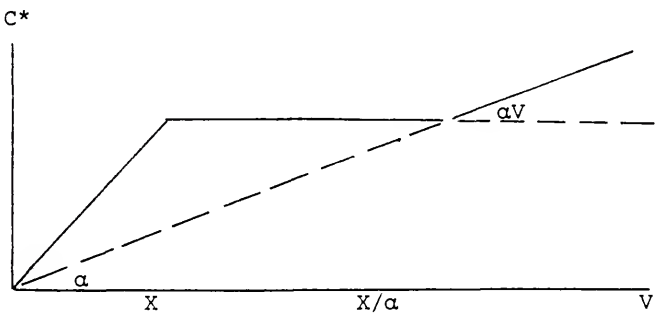
$$C = \alpha V e^{-\delta T_N} \frac{\ln(\alpha V/X) + (r-\delta+\sigma^2/2)\,T}{\sigma\sqrt{T}} - e^{-rT_{XN}} \frac{\ln(\alpha V/X) + (r-\delta-\sigma^2/2)\,T}{\sigma\sqrt{T}} \qquad (4.16)$$

where α stands for the lender's percent of ownership upon conversion.

The graph below illustrates the relationship between the call price and the conversion value, given the exercise price, the time to maturity, and the riskless rate. The Black-Scholes call option price lies below the maximum possible value, $C = \alpha V$ (except where $\alpha V = 0$), and above the minimum value, $C = \max[0, \alpha V - X\exp(-rT)]$.



As the conversion value is a fixed fraction α of the property value (the original loan to value ratio) the equivalence between the conversion element of the convertible mortgage and the convertible debt is easily obtained. The following graph shows the payoff function at time T*, the expiration date of the option:



Consider only the situation where V* is larger than X, as the case where V* is smaller than X (the default risk of the loan) has already been dealt with when the collateralized loan was priced. When V is larger than X/α, the loan

would be converted and the lender obtains fraction (α) of the property value.

This payoff represents the option payoff. Call this payoff C, as indicated before, and define it as

$$C^* = \text{Max}(\alpha V^* - X, 0)$$

where * stands for the value at time T.

The solution can be written in general form as

$$C = C(\alpha V, X, T, \sigma^2, r, \delta),$$

where

$$\frac{\partial C}{\partial \alpha V} > 0; \quad \frac{\partial C}{\partial \sigma^2} > 0; \quad \frac{\partial C}{\partial x} > 0; \quad \frac{\partial C}{\partial r} > 0; \quad \frac{\partial C}{\partial T} > 0; \quad \frac{\partial C}{\partial \delta} > 0.$$

The partial effects have intuitive interpretations. As the property value increases, the expected payoff of the option also increases. Payoffs decrease with higher exercise prices. The longer the time to maturity or the higher the interest rate, the greater the value of the option because the present value of the exercise payment is lower. Also, with larger variance rates on the property value or longer times to maturity the probability of a large price change in the option feature during the life of the option grows. Since the call price cannot be negative, a larger range of possible property values increases the

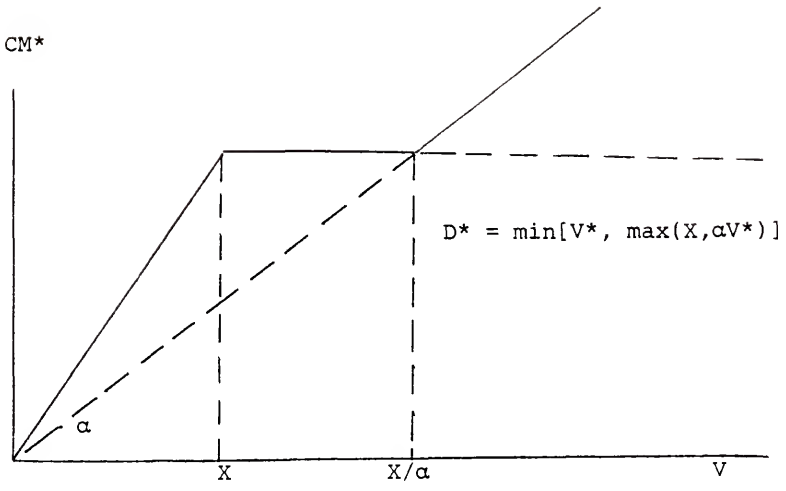
maximum value of the option feature without lowering the minimum value.

Recapitulation

Now that the elements of a convertible mortgage have been priced, the option pricing model for convertible mortgages can be stated. The value of a convertible mortgage (CM) is as follows:

$$
\begin{aligned}
CM &= D + P + C \\
&= Ve^{-\delta T_N}\left(\frac{-\ln (V/X) - (r-\delta+\sigma^2/2)\ T}{\sigma\sqrt{T}}\right) \\
&\quad + e^{-rt_{XN}}\left(\frac{\ln (V/X) + (r-\delta-\sigma^2/2\ T}{\sigma\sqrt{T}}\right) \qquad (4.17) \\
&\quad + \alpha V\ (1-e^{-\delta T}) \\
&= \alpha Ve^{-\delta T_N}\ \frac{\ln (\alpha V/X) + (r-\delta+\sigma^2/2)\ T}{\sigma\sqrt{T}} \\
&\quad -e^{-rT_{XN}}\ \frac{\ln (\alpha V/X) + (r-\delta-\sigma^2/2)\ T}{\sigma\sqrt{T}}
\end{aligned}
$$

The graph below illustrates the payoff function to a convertible mortgage at maturity as a function of the value of the property.



If the value of the property is less than the payment X due the convertible debtholders, then the value of the convertible mortgage is the value of the property, since, like ordinary mortgage loans, the lender has the right to the property if the debt payment is not received. If the value of the property is such that the conversion value (αV) is greater than the debt payment, the option will be exercised. If the lender does not exercise the option at the end of the period, the lender receives either the debt payment X or the value of the property, whichever is less. The convertible will only be exercised if it is worth more exercised than not. At maturity the value of the convertible will be

$$CM\ (V,0) = V \text{ if } 0 \leq V \leq X$$

$$= X \text{ if } X \leq V \leq V^*$$

$$= \alpha\ V \text{ if } V^* \leq V.$$

where $V^* = x/\alpha$

A convertible mortgage is obviously equivalent to a nonconvertible mortgage, D, plus a European call option, C, plus the present worth of participation income:

$$CM\ (V,X,T,\alpha,\sigma^2,r) = D\ (V,X,T,\sigma^2,r,\delta) + C\ (\alpha V,X,T,\sigma^2,r,\delta) + P(\alpha V,T,\sigma^2,\delta)$$

where

$$\frac{\partial CM}{\partial V} \equiv \frac{\partial D}{\partial V} + \alpha\ \frac{\partial C}{\partial \alpha V} + \frac{\partial P}{\partial \alpha V} > 0,$$

$$\frac{\partial CM}{\partial \alpha} \equiv V\ \frac{\partial C}{\partial (\alpha V)} > 0,$$

$$\frac{\partial CM}{\partial X} \equiv \frac{\partial D}{\partial X} + \frac{\partial C}{\partial X} > 0,$$

$$\frac{\partial CM}{\partial T} \equiv \frac{\partial D}{\partial T} + \frac{\partial C}{\partial T} + \frac{\partial P}{\partial T} <> 0,$$

$$\frac{\partial CM}{\partial \sigma^2} \equiv \frac{\partial D}{\partial \sigma^2} + \frac{\partial C}{\partial \sigma^2} + \frac{\partial P}{\partial \sigma^2} <> 0,$$

$$\frac{\partial CM}{\partial r} \equiv \frac{\partial D}{\partial r} + \frac{\partial C}{\partial r} < 0.$$

$$\frac{\partial CM}{\partial \delta} \equiv \frac{\partial D}{\partial \delta} + \frac{\partial C}{\partial \delta} + \frac{\partial P}{\partial \delta} <> 0.$$

The convertible mortgage is clearly a combination of debt secured by the property, a participation yield, and an

option on the equity in the property. The price of the convertible will thus reflect all of these features. If the value of the property, V, increases, all elements of the convertible become more valuable. If the fraction of the property received through conversion, α, increases, then the call portion of the convertible mortgage becomes more valuable without reducing the debt portion. If the face value of the debt increases, the payoff to the convertible increases. If either the time for maturity or the variance rate increases, the debt portion becomes less valuable while the call portion becomes more valuable. Either effect can dominate. If the interest rate increases, the present value of the promised repayment is reduced.

Application of the Model: A Case Example

Appendix II provides a user-interactive program, written in IFPS (Interactive Financial Planning System), that may be used to calculate the value of each element of a convertible mortgage. This suggested computer program allows the user to readily evaluate a convertible mortgage using the basic option pricing model developed in this chapter.

The program requires the user to input the following variables:

1. V, the initial property value

2. X, the face value of debt
3. T, the time until conversion
4. δ, the participation yield = $\left(\frac{\text{participation income}}{V}\right)$[1]
5. σ, the standard deviation of the property value movement[2]
6. α, the percentage of V lender receives upon conversion[3]
7. r, the riskless rate.[4]

[1] δ = P1 in the program

[2] σ = SIGMA in the program

[3] α = P2 in the program

[4] r = R in the program

The program provides the following output:

1. D, the present worth of debt
2. P, the present worth of participation income
3. C, the present worth of the option to convert
4. CM, the present worth of the convertible mortgage

Input/Output

To illustrate the operation of the option pricing model a convertible mortgage has been valued for the particular set of parameter values given in Table 5. The example is similar to the convertible mortgage valued in the preceding chapter. This example is not, however, exactly like the case presented in Chapter 4. The models require different

input, have unique underlying assumptions, and use far different techniques to determine value. There has been an attempt to make the two examples as similar as possible. Both represent $8,000,000 loans on properties valued at $10,000,000. The same coupon rate, conversion date and conversion option are examined.

Interpretation of Output

Data from Table 5 suggest that the convertible mortgage being analyzed is an attractive investment. The present value of the convertible mortgage is estimated to be $8,931,126. This compares favorably with the loan amount of $8,000,000.

Table 5. Parameters of Case Example

Input Variable	Value
Value of Property, V	$10,000,000
Face Value of Debt, D	8,000,000
Time Until Conversion, T	5 years
Participation Yield, δ	.02
Standard Deviation, σ	.10
Participation/Conversion Rate, α	.80
Riskless Rate, r	.09

Table 6. Convertible Mortgage Valuation

Output	Value
Value of Debt, D	$ 7,907,370
Value of Participation Income, P	761,301
Value of Conversion Option, C	262,455
Value of Convertible Mortgage, CM	8,931,126

The debt element of the convertible would not by itself constitute an attractive investment position. The estimated value of interest payments ($7,907,370) is less than the original loan amount. The other two elements of the loan more than make up for this deficiency however. Participation income is valued at $761,301 and the option to convert at the end of the fifth year is believed to be worth $262,455. Based upon the assumptions specified in this example the option pricing model evaluates this loan contract favorably.

Sensitivity Analysis

One of the attractive features of convertible mortgages is their flexibility. If the lender or borrower found the terms of the mortgage represented in the example unacceptable, the impact of a change to the contract can easily be evaluated. The model may be subjected to sensitivity analysis to show what effect a change would have on any of the elements of the convertible mortgage. Table 7 shows some representative results of changing each of the input variables in the model. In the table various input parameters are changed one at a time with all other variables being held constant at the standard values assigned them in the example just cited. Table 8 shows the direction of the impact on each convertible cash flow given an increase in a given variable.

Interpretation of the sensitivity analysis is in most cases intuitive. As the coupon rate on debt increases, the value of the debt element also increases. Increased coupon rates do not affect the participation returns, but they do have a negative impact upon the option element (C). The negative influence on C results from asset value reduction at conversion due to higher interest payments. Increases in the coupon rates impact favorably on the estimated value of the total package (D + P + C). Participation income often replaces lost interest income when convertibles are originated at discounted coupon rates. Two variables relate to the contract terms of participation. The participation rate, α, determines what percentage of income is subject to participation, or, in the event of conversion, what percentage of the property will be owned by the lender. If α increases, there is no impact upon debt, however all other elements are valued higher. If the actual yield at which the lender receives participation income, δ, increases the results are quite different. If the lender receives a higher percentage of income as his participation yield, the effect is to lessen the value of debt and the conversion option. This occurs since the default risk rises on debt as monies are taken out of the property. Also the terminal value of the property is reduced by any payout. The overall impact on the mortgage's value was positive for the yield range tested however, since the increased participation income dominated the losses in D & C.

Table 7. Sensitivity Analysis

Variable	Value			
	Debt	Participation Income	Conversion Option	Convertible Mortgage
X-Face Value of Debt (relates to coupon int. rate)				
$12,884,080 (10% coupon rate)	$7,793,134	$ 761,301	$ 306,361	$8,860,795
$13,179,574 (10.5% coupon rate) (std.)	7,907,370	761,301	262,455	8,931,126
$13,480,465 (11% coupon rate)	8,016,081	761,301	223,443	9,000,824
α-Participation Rate				
.60	7,907,370	570,975	14,377	8,492,723
.70	7,907,370	666,138	79,739	8,653,247
.80 (std.)	7,907,370	761,301	262,455	8,931,126
δ-Participation Income Yield				
.015	7,978,141	578,052	319,013	8,875,206
.02 (std.)	7,907,370	761,301	262,455	8,931,126
.025	7,829,250	940,025	214,099	8,983,373
r-Riskless Rate				
.08	8,142,960	761,301	182,013	9,086,274
.09 (std.)	7,907,370	761,301	262,455	8,931,126
.10	7,649,515	761,301	365,821	8,776,637
T-Time Until Conversion				
5 (std.)	7,907,370	761,301	262,455	8,931,126
7	7,724,207	1,045,134	248,938	9,018,280
10	7,408,743	1,450,154	221,150	9,080,047
σ (sigma)				
.05	8,255,030	761,301	36,824	9,053,155
.075	8,087,287	761,301	133,644	8,982,232
.10 (std.)	7,907,370	761,301	262,455	8,931,126
.1250	7,722,673	761,301	406,867	8,890,841
.15	7,535,941	761,301	559,482	8,856,723
.25	6,790,423	761,301	1,196,982	8,748,706

Table 8. Sensitivity Analysis Dynamics

As Variable Increases	Effect on Value*			
	Debt	Participation Income	Conversion Option	Convertible Mortgage
X-Face Value of Debt	+	/	-	+
α-Participation Rate	/	+	+	+
δ-Participation Income Yield	-	+	-	+
r-Riskless Rate	-	/	+	-
T-Time Until Conversion	-	+	-	+
σ-Sigma	-	/	+	-

* +, value increases
-, value declines
/, value is unchanged

Note: The above dynamics are based upon all other variables being held constant at the standard.

The variables discussed thus far have related to contract terms dealing with the debt or participation portion of the loan. The choice of a conversion date impacts upon the option to exchange a lending position for an equity position. The model only evaluates the possibility of conversion occurring once. The fact that there are actually multiple conversion dates is addressed later in this chapter. As T (time until conversion) increases, both the value of D and C decrease. The value of P increases greatly, and the effect on CM nets out as a small gain. If the riskless rate is increased, values for all elements except P decline. Finally, as the variance of the property's value increases, the value of CM decreases.

The greater the risk of possible property values (higher and lower) the greater the probability becomes of default on the debt. The value of D decreases greatly as σ increases. Conversely, the value of C increases greatly, as any option does, when the variance in property value rises. Changes in σ have no affect on P.

Practical Application of Option Pricing Theory to Convertible Mortgage Valuation

Based on the model presented in the previous sections, it is theoretically possible to value convertible mortgages using option pricing theory. Is, however, application of the devised model a worthwhile endeavor? This question can only be answered by careful investigation of the important assumptions underlying option pricing theory and analysis of the adjustments required to make the formula meet "real world" behavior. Practical application of the derived convertible mortgage formula will occur only if the assumptions are not violated and the adjustments correctly extend the underlying theory. The remainder of this section discusses required assumptions, estimation of inputs, and other complicating factors which may occur due to the financial environment in which convertible mortgages are originated and traded.

Underlying Assumptions

In deriving their model, Black and Scholes employ the following assumptions:

1. There are no penalties for short sales.
2. Transaction costs and taxes are zero.
3. The market operates continuously.
4. The risk-free interest rate is constant.
5. The stock price is continuous.
6. The stock pays no dividends.
7. The option can only be exercised at the terminal date of the contract.

Subsequent modification of the basic Black-Scholes model by Merton (1973, 1974, 1976) and others shows that the analysis is quite robust with respect to relaxation of the basic assumptions under which the model is derived. No single assumption seems crucial to the analysis. Thorpe (1973) examines the effects of restrictions against the use of the proceeds of short sales. Ingersoll (1976) takes explicit consideration of the effect of differential taxes on capital gains and ordinary income. Merton (1976) argues that the continuous trading solution approximates the asymptotic limit of the discrete trading solution when the stock price movement is continuous. Merton (1973) also generalizes the model to the case of a stochastic interest rate. Thus, it appears that the relaxation of the first four assumptions involving the specification of the behavior of the capital market environment modifies the analysis in no significant way.

In addition, the analytical techniques developed by Black and Scholes remain valid, even if the last three assumptions dealing with the specification of the stock and option are relaxed. Cox and Ross (1976) successfully employ a Black-Scholes type analysis to examine a case in which stock price movements are discontinuous. Their discrete-time logarithmic model, by reaching the same option pricing conclusion as a hedging model, indicates the robustness of the Black-Scholes formula to its assumption of continuous trading.

Even if an investor, for some reason, cannot implement a dynamic riskless hedging strategy similar to that described in the first section of this chapter, the investor may very well value an option according to the Black-Scholes formula. Merton (1973) and Thorpe (1973) modify the model to account for dividend payments on the underlying stock. Finally, Merton (1973) shows that the Black-Scholes solution for an option which can be exercised only at maturity can be appropriate to value a call option which may be exercised prior to the maturity date.

Input Variables

Black and Scholes derive the solution to the option pricing problem as a function of only five variables:

1. the stock price,
2. the exercise price of the option,
3. the time to maturity of the option,

4. the risk-free interest rate,

5. the variance rate on the stock.

Before considering how each variable is estimated, a word about what the formula does not depend on is deemed prudent. In terms of practical usefulness, what the formula does not depend on is almost as important as what it does depend on. In particular, the formula does not depend on an assessment of the future or expected stock (property) price. Also, it does not depend on investors' attitudes toward risk. Since these are not observable, any formula that required them as inputs would be less useful. The fact that the option formula is independent of expectations and other subjective measures bodes well for its applicability.

An important point for tests and applications of the model is that all of the variables except the variance are directly observable. The first three variables are particularly easy to determine. In the convertible mortgage model the initial property value replaces the stock price, the exercise price becomes the outstanding loan balance, and the time to maturity is the time remaining until a conversion privilege occurs. All of these variables are readily observable. The method of estimating the interest rate and the volatility of the property's value will be discussed next.

The Black-Scholes option model requires an estimation of interest rates. This variable is not easily observable.

The interest rate should measure the risk-free borrowing and lending rate over the period of the option. This rate can be obtained by using the yields of U.S. Treasury instruments which have maturities that correspond to the convertible mortgage conversion date. The typical convertible mortgage first converts in the fifth year; therefore, a Treasury instrument with a five year maturity can be used to estimate the appropriate interest rate. Jarrow and Rudd (1983) and Cox and Rubinstein (1985) show how to get these figures from the financial pages.

A definite problem that remains however is estimation of variance with regard to property value. Two recent approaches to this problem provide insight. Fogler, Graneto, and Smith in the Journal of Finance (July 1985) show that estimates of variance can be approximated by use of relevant real estate market indexes. The estimate for variance would be:

$$\sigma^2 = \frac{1}{N-1} \sum_{k=1}^{n} ((\ln(R_k))^2 - \mu^2)$$

$$\mu = \frac{1}{N} \sum_{k=1}^{n} \ln (R_k)$$

where R_k is the ratio of the index_t over the index_{t-1} and μ is the estimate of the mean rate of change.

The second approach is more heuristic but proves valuable if sensitivity analysis is to be applied. A binomial process can express movements of the asset price in terms of up and down parameters, u and d, and the probabilities q and 1 - q. These parameters (u, d, q) implicitly define the variance and return that was assumed in the option pricing model for convertible mortgages. Cox and Rubinstein (1985) show that

$$E[\ln(S^*/S)] - [q\ln(u/d) + \ln d]n = \mu n$$

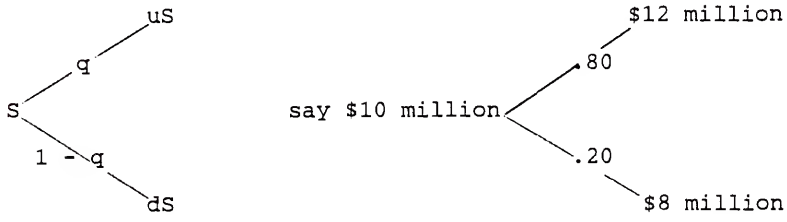
$$Var[\ln(S^*/S)] = q(1-q)[\ln(u/d)]^2 n = \sigma^2 n$$

where n = the number of periods

S = asset price

So, if one is able to construct a binomial tree with u, d, and q depicting their beliefs about the values of the property at the end of the period, they can derive an estimate of the variance.

For example,



Assume n = 5. Then

$$\mu = [.8*\ln(12/8) + \ln(8)]5 = 12.017$$

$$\sigma^2 = .8*.2[\ln(12/8)]^2*5 = .1315$$

Other Complicating Factors

The Black-Scholes formula was derived under a set of restrictive assumptions. Where possible, one needs to adjust the formula when these assumptions are not met. Adjustments can usually be made, and where such adjustments are not possible, we need to weigh the effect of the assumptions on the accuracy and profit potential of the formula. There are several impacting factors which deserve attention when the usefulness of the model developed is contemplated. These factors include: multiple conversion dates, default risk timing, and overlapping claims of the lender and borrower on the property.

Multiple Conversion Dates

Valuing convertible mortgages is complicated by the fact that conversion typically can occur at different points in time. Multiple conversion dates force the mortgagee to decide whether to convert when the right is first offered or to let that option expire and convert at a later conversion date. This situation creates a compound option.

Consider a typical convertible mortgage which allows conversion at the end of the fifth year or in the seventh year. So, at the fifth year we really choose

	Payoff	
Exercise now	$\alpha V_5 - X$	if $\alpha V_5 - X > C_7$
Wait till 7th year	0	if $\alpha V_5 - X < C_7$

where C_7 is the present value (at year 5) of the option to exercise at year 7.

Geske (1979) priced a somewhat similar compound option. Applying his technique results in the following formula:

$$C = e^{-s*T}V*N_2(x,y;(t/T)\cdot^5) - e^{-r*T}\alpha V*N_2(x-\sigma*t\cdot^5,$$

$$y-\sigma*T\cdot^5;(t/T)\cdot^5) - e^{-r*T}\alpha V*N(x-\sigma*t\cdot^5)$$

where $$x = \frac{\ln(V/\bar{V}) + (r-s+\sigma^2/2)*t}{\sigma*t\cdot^5}$$

$$y = \frac{\ln(V/\alpha V) + (r-s+\sigma^2/2)*T}{\sigma*T\cdot^5}$$

N_2 is the bivariate normal distribution with three inputs x, y, and z.

V_5 = property value at year 5

C = current value of the compound option

t = the time to the first option maturity (5)

T = time to second option maturity (7) and $\bar{V}$ satisfies

$$e^{-s*(T-t)}*\bar{V}*N(z) - aV*e^{-r*(T-t)}*N(\sigma*(T-t)^{.5}-aV = 0$$

where $$z = \frac{\ln(\bar{V}/V) + (r-s+\sigma^2/2)*(T-t)}{\sigma * (T-t)^{.5}}$$

Iterative methods must be used to find $\bar{V}$; therefore, it does not seem practical to calculate this compound option for convertible mortgages. The relevance of the compound option feature seems small. One can use the formula for C at a conversion date to determine whether exercising the option or waiting until the next option date is the best strategy. If conversion can occur at the end of years 5, 7, or 10, the first choice would be C_7 versus $aV_5 - X$. If conversion did not occur in year five, the same choice would occur in year seven, with C_{10} versus $aV_7 - X$.

The suggested technique for deciding whether or not to exercise the conversion option is quite simple. The decision is made based upon future expectations on each conversion date. This technique, however, does not solve the problem that multiple conversion dates create for convertible mortgage valuation. If convertible mortgages are valued based upon conversion occurring at the earliest time possible, it is obvious that the value of remaining

conversion options are not accounted for. A live option has value. The valuation model developed earlier allows the investor to value the mortgage based upon expected conversion at any date. In practice, the investor will convert only if exercising that privilege appears more favorable than extending the option.

Default Risk/Coupon Payments

The Black-Scholes model was adjusted for participation payouts to incorporate the additional default risk this action causes. This adjustment, however, does not fully evaluate the default risk associated with a coupon paying instrument. The formula developed does not deal specifically with the consequences of a missed coupon. The basic model only evaluates the possibility of default on a conversion date, as the debt is structured as though it were a discount loan. In reality, convertibles do require periodic payments, and if a coupon payment is missed the loan may be worthless, or at least subject to default. The basic model fails to assess default risk as it occurs. Obviously the informational content associated with successful payment of each coupon is lost in the model.

Considerable thought was given to the possibility of valuing coupon-paying convertibles in a more sophisticated manner. Coupon payments and their effect on default risk, and the ultimate value of a convertible mortgage were carefully considered. Intuition suggested an attempt to

value coupons as they occurred would improve the model. Previous approaches to this issue were studied with no clear, and cost-effective, answer arising. After careful reconsideration of the assumptions underlying the issue it was found that a more complex model probably was not required. However, before discussing the important underlying assumption a brief review of alternative methods used to value coupon payments will be given.

The effect of intermediate coupon payments is to transform the borrower's interest into a compound option. At every coupon date the borrower has the option of either buying the next option by paying the coupon or forfeiting the property collateralized by the loan to the lender. The final option is to repurchase the lender's claim on the property by paying both the principal and the final coupon.

One reasonable approach to valuation of the coupon loan is to model the debt as a portfolio of risky discount loans of differing maturities where the face value of one of the loans is equal to the coupon (or principal) payable at its maturity. This idea suggests that the debt portion of a convertible mortgage can be valued as a portfolio of riskless discount loans less a portfolio of puts.

A portfolio of puts is subtracted since the discount loan portfolio formula contains a call option to repurchase the property on maturity. The coupon payments do not actually create the situation where repurchase is possible

on each payment date; each payment merely buys another option that will eventually (at maturity) allow for the repurchase of the collateralized property.

Robert Geske (1977, 1979) presented a complex theory for pricing options on options. Geske (1977) demonstrated that an analytic solution could be obtained for valuing compound options in either discrete or continuous time and showed that this approach introduced capital-structure effects into the pricing of call options. In his 1979 work a formula was derived for valuing subordinated debt as a compound option. Geske's expression for the value of a risky coupon bond is found by recursively solving for the values at each boundary encountered in terms of the immediate solution to the previous boundary.

Both the portfolio of risky loans idea and the Geske approach to valuation of compound options have weaknesses when applied to convertible mortgage valuation. The portfolio model does allow for different discount rates to be applied to each coupon payments. The model does not, however, specifically identify default risk as it occurs. Each coupon payment is valued as if independent of all other payments. It should be noted, however, that if any loan in the portfolio defaults at any given time, then all succeeding discount loans in the portfolio would actually be worthless. If a constant discount rate were applied to all loans in the portfolio, and that same rate was used in the

basic model, both approaches would yield identical answers. The portfolio technique also requires that, V, from the Black-Scholes model be calculated for each distinct loan. This value is not readily available and a proxy of it might be found from developing yield curves that relate to convertible mortgages. This proxy of V, while plausible, is not an altogether appropriate substitute.

The Geske technique can only be solved by iterative methods to find $\overline{V}$, which is the value of V which solves the integral equation $S_i(V) - X_i = 0$. The integral equation is analytically important since the firm is considered bankrupt whenever $S_t(V) < X_t$. Aside from the exhaustive quantitative requirements of the Geske equation, the model also has a restrictive assumption that reduces its value. The model allows no payouts (dividends or participation income) other than coupon interest payments. This assumption is obviously violated in most convertible mortgages.

The entire matter of compound options may actually be irrelevant due to the marketability of most real estate assets. Assume bankruptcy were to occur due to default on a coupon payment, and that debtholders forced the firm into receivership and attempted to gain control of the property. If such a transition could occur without in any way disrupting the activities of the property--its revenues or its costs--there would be little presumption that the value of the assets would be influenced by the risk of ruin. To be

sure, management changes would occur which might alter the fortunes of the property, but generally the total worth of the project could not differ significantly from its value before default occurred. Real estate projects go bankrupt frequently, yet continue to operate with seemingly little discontinuity.

Bankruptcy costs are introduced by the assumption that the productive assets of the firm are sold in imperfect secondary markets. This implies that the liquidation value of the firm's assets is always less than the market value of a well-managed nonbankrupt firm (Scott, 1976). Bankrupt real estate properties typically sell at prevailing market prices unlike many corporate assets (machinery and inventory) which quite likely are not salable at market value. The compound option issue is only an issue if default occurs before loan maturity and there are bankruptcy costs associated with the default. The assumption that such costs are not associated with real estate insolvency is not particularly farfetched.

It might also be added that much of the concern about default risk associated with convertible mortgages is effectively eliminated when the loan is originated. Mortgages are secured debt. The fact that the convertible is a collateralized loan reduces the uncertainty about the nature of the risk of that portion of the project, since the collateralized assets cannot be disposed of without the

permission of the mortgagee. If the underlying collateral property were properly appraised at the time the loan was made, even though the loan to value ratio may be high, the degree of default risk assumed should be acceptable.

Overlapping Claims

Option pricing theory valuation of debt instruments suggest that a loan is equivalent to a contract where the assets of the firm (the property) are sold to the debtholder by the borrower for the proceeds of the loan plus a call option to repurchase the assets from the debtholder at loan maturity with an exercise price equal to the face value of the debt. Do borrowers actually have a right to repurchase the assets they pledge as collateral? If the debt were a corporate bond or a standard collateralized mortgage the answer would be simple--yes. At loan maturity if the borrower has successfully met coupon payments and the principal is repaid, the lender's claim on the collateralized assets expires. The answer is not so simple for convertible loan contracts where the lender has bought the right to convert a debt position into one of ownership.

Let us examine a convertible mortgage contract. Assume that prepayment of the loan is not allowed. It can also be assumed that the borrower and the lender will act in their own best interest. If the borrower has the opportunity, he will reclaim the property if its value exceeds that of the loan. If, however, the loan amount is greater than the

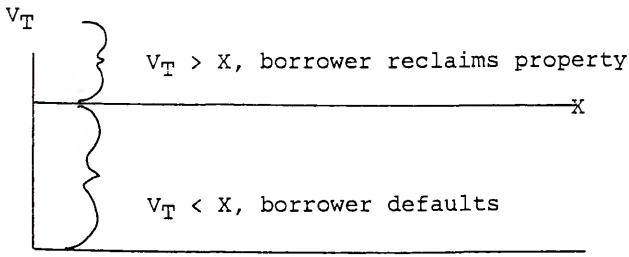
property's value, it can be assumed that the borrower will default on the loan and surrender ownership of the property. If the lender has the option of receiving the loan repayment or the property, his choice will depend upon the value of his proportional interest in the property (its conversion value) versus the amount owed on the loan. The lender will take whichever asset is more valuable.

Convertible mortgages are written in many different ways. It is possible that the lender's right(s) to convert may expire before the loan actually matures. Another possibility is that the last option privilege occurs on the date of loan maturity. Depending on which of these situations exist, the action of the parties-at-interest will differ at loan maturity. Obviously if the lender exercises the right to convert before loan maturity, there is no action that the borrower can take other than surrendering the contractual portion of ownership agreed upon.

Assuming conversion has not occurred, let's consider what action (reaction) the parties might take at loan maturity date under the two situations mentioned previously. The first situation is one where the last conversion option occurs before the loan matures. In this case the borrower decides how the loan contract is to be resolved. If the value of the property at loan maturity, V_T, is greater than the loan balance, X, the property will be reclaimed. If V_T is less than X, the borrower will default on the loan and

the lender receives the property. The following graph shows what decision will be made.

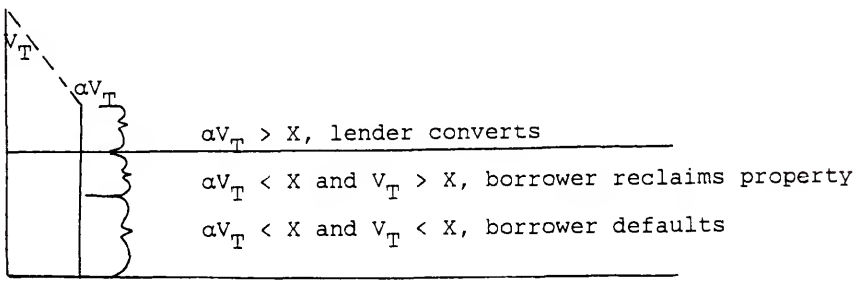
Situation A: Conversion Date(s) Expire Before Loan Maturity



A more interesting situation exists when the lender does have the right to convert on the date the loan matures. In this case the lender has the right to convert, and this will occur if the conversion value, αV, on that date exceeds the loan balance. Conversion value equals current property value times the percentage amount of the property due the lender upon conversion. If αV_T is less than X, the lender will not convert; however, he may still end up owning the property. If the option to convert is not exercised, it is the borrower who ultimately decides what will occur. The borrower will default on the loan if $V_T < X$. This action was true in the earlier situation also. There is a circumstance, however, where the lender will not convert yet the borrower will not default. If the property value is greater than the loan balance, default will not occur. The lender's

conversion value is less than the loan balance so conversion does not occur, yet the property value is higher than the loan balance so the borrower reclaims the property. This situation is portrayed in the following graph.

Situation B: Last Conversion Date Coincides with Loan Maturity Date



It is apparent that the borrower's right to repurchase the underlying assets are restricted in convertible mortgage contracts. If the last conversion option occurs before the loan maturity date, the mortgagor's right to repurchase the collateral property exists only after all conversion options have expired. The borrower maintains this right by continuing to make scheduled coupon payments until loan maturity. In the situation where the mortgagee has a conversion option on the date the loan matures, the mortgagor really doesn't have first refusal of reclaiming the property. The lender decides whether to take the property or the loan balance. If the lender opts for the loan balance, the borrower reacts by either making the requested payment or defaulting and relinquishing the

property to the lender. The right to repurchase the property is conditional upon the lender's action. Remember, however, that the lender paid a premium for the right to have the privilege to convert.

Summary

Convertible mortgages are complex securities and their valuation is a difficult task. This chapter has attempted to apply option pricing theory to the valuation of convertibles. The Black-Scholes option pricing model was reviewed in order to outline the theory. The formula determines the option price that is necessary to eliminate the possibility of profit opportunities. Since many financial instruments can be considered as options, the theory has application to a wide range of financial instruments.

Option pricing theory was applied to convertible mortgages as a valuation method. By reviewing and extending work of others in this area, a model was developed to value convertibles. The model was programmed for computer use and sensitivity analysis over a plausible range of values was conducted. The model does appear to approximate value in a realistic manner. Regrettably, few actual data on convertibles are available to test the model against.

Can the model be applied in a practical manner to valuation of convertible mortgages? The answer is still unclear. Most of the seemingly restrictive assumptions of

the Black-Scholes formula can be dropped. Correctly calculating some required input variables remains a difficult task. Estimating the appropriate variance rate of a real estate project is particularly troublesome. How to deal properly with multiple conversion dates is another problem. Lastly, does the formula effectively evaluate the default risk inherent in this type of financial contract?

The importance of this chapter is not that "the" correct option pricing model was developed; but rather that the issues surrounding use of this body of knowledge were carefully considered and the model reflects that thought.

CHAPTER FIVE
THE MARKETPLACE--CONVERTIBLE MORTGAGE QUESTIONNAIRE

Introduction

This chapter provides an insight into institutional investment in convertible mortgages. By use of a comprehensive questionnaire, data were collected from a diverse group of the nation's largest institutional investors in real estate. Careful analysis of the data gathered provides a better understanding of how and why convertible mortgages are being written. This research provides insight into the industry's perception of convertibles, the common features of the instruments being written, and the manner in which these securities are currently being priced.

Another use of the data collected relates to testing the valuation models developed in chapters three and four. Some respondents provided illustrative examples of representative convertible mortgages. Actual mortgage contracts have also been provided. This information allows the valuation models to be tested against actual (or realistic) convertible mortgage data. This application is described in Chapter Six and provides clues as to the usefulness of the valuation techniques that have been developed.

Research Methodology

Sample Group

This research consists of a questionnaire about current institutional investment in convertible mortgages. It was sent to the 50 largest American institutional investors in real estate. The list of investors was compiled from an article in Institutional Investor titled: "The 1984 Institutional Investor 300-Ranking America's Top Money Managers." Firms were ranked according to total real estate under management; this included both mortgages and equity real estate. It should be noted that mortgage-backed securities, such as Ginnie Mae pass-throughs and Freddie Macs, were listed under fixed-income securities and therefore not included for ranking purposes.

The total value of real estate managed by these firms in 1983 was slightly more than $200 billion. Twenty-three of the fifty firms surveyed were life insurance companies. The next largest group surveyed were real estate investment advisors/managers (10). Seven pension funds, four commercial banks, four savings and loans, one trust company, and one savings bank completed the list of those who were mailed questionnaires.

Prior to mailing the questionnaire and cover letter telephone contact was made with each firm. This procedure aided in identifying the proper individual to whom the survey would be sent. Approximately one month after the

questionnaire had been mailed, a followup telephone call was directed to those firms who had not responded to the questionnaire. It is believed that these procedures helped produce a satisfactory survey response from a rather small sample. Thirty-six questionnaires were completed for a 72 percent response rate.

The Questionnaire

The questionnaire consists of five sections which gather the following data:

1. Who invests in convertible mortgages?
2. When and why does a firm invest in convertible mortgages?
3. What are the representative characteristics of convertible mortgages?
4. How do investors price convertible mortgages?
5. What are the specifics of an actual convertible mortgage your firm has invested in?

A copy of the complete questionnaire is included in Appendix III.

Results

Section I--Background Information

This portion of the survey gathered general background information on each respondent and determined whether or not the firm was presently making convertible

mortgage commitments. The background data have already been reviewed in the section titled Sample Group.

Question five in Section I of the questionnaire asks--Does your firm invest in convertible mortgages? Twenty-two percent of the respondents (8) answered that they did invest in this type of mortgage. Three pension funds, two commercial banks, two life insurance companies, and one real estate investment adviser were the institutional investors who responded to the survey and were also originating convertible mortgages. Those investors who were not making convertible mortgage commitments were asked whether they expected to do so within the next two years. Eighty-two percent responded no; the remainder were uncertain.

Why haven't institutional investors made convertible mortgage investments? Respondents who did not invest in convertible mortgages were asked why they had not. The questionnaire identified three possible reasons and asked the respondents to specify others. Table 9 shows the reasons cited for not investing in convertible mortgages.

Table 9. Reasons for Not Investing in Convertible Mortgages.*

Reason	% of Responses	% of Total Responses
Question legal ramifications	9	27
Do not believe they are attractive investment	5	15
Limited knowledge about convertible mortgages	3	9
Other, please specify+	16	49

*Respondents were instructed to circle all reasons that were appropriate.

+Listed below are briefs of "other" reasons

- Not requested by clients
- Only make equity investments
  As residential mortgage lender they can't own property
- Other investment combinations are better
- Risk too great for clients
- Doesn't fit lending specialty
- Do not trust developer to maintain property
- Prefer joint ventures
- Prefer equity control from start
- Outside of present investment realm
- Do not like risk/reward potential
- Tax ramifications
- Doesn't fit product needs
- Limited equity activity
- Can lead to unsound mortgage investment due to high L/V ratio
- Very few opportunities
- Better ways to structure equity investment

Section II--Convertible Mortgage Commitments

The remaining parts of the questionnaire were answered only by those institutional investors who invest in convertible mortgages. They were first asked why they make investments in convertible mortgages. Table 10 outlines their reasons.

Table 10. Reasons for Making Convertible Mortgage Investments.*

Reason	# of Responses	% of Total Responses
Viewed as effective inflation hedge	3	16
Favorable returns based on risk taken	7	37
Provide investment diversification	3	16
Other, please specify+	6	31

*Respondents were instructed to circle all reasons that were appropriate.

+Listed below are briefs of "other reasons"

- Like owner/developer looking after property on daily basis
- Benefit split is positive feature
- Capitalizes on current attractive tax environment
- Way to split tax benefits
- Often CMS are the only way to get deal
- Seller gets tax benefits, while investor gets higher current return than on equity purchase

When asked under what circumstances they preferred convertible mortgages to conventional fixed-rate mortgages, the institutional investors gave a variety of answers. Two respondents generally preferred convertibles over fixed-rate

mortgages at all times. Other investors stated that they consider convertibles to be equity investments or that sometimes they were not looking for traditional fixed-rate obligations but rather investments with more flexibility. Another institutional investor does not buy fixed-rate mortgages as long-term investments and typically uses corporate bonds as the primary income-producing investment vehicle. Two investors stated that convertible mortgages are a good way to trade tax benefits to the developer for higher returns than fixed-rate mortgages could provide.

How risky do institutional investors think convertible mortgages are? When asked--does your firm view convertible mortgages as more or less risky than conventional fixed-rate mortgages--sixty-three percent felt that the convertibles were more risky. The reasons given for considering convertibles more risky were

- Lower coupon rate, higher loan to value ratio, possible bankruptcy, uncertainty of conversion right
- Return is tied to success of real estate project
- Higher loan to value ratio with lower coupon rate
- Uncertainty of future property value
- Seller may not maintain property, legality of conversion right, lower coupon rate, higher loan to value ratio

Those who believe that the convertible instruments are less risky than conventional fixed-rate mortgages gave these reasons

- Equity feature provides inflation hedge
- Foreclosure position can often be avoided by conversion; this produces legal cost savings
- Lower price risk due to conversion feature.

The last two questions in this section of the questionnaire gather data on dollar commitments in convertibles and investor satisfaction with this type of investment. The dollars invested in convertible mortgages ranged from a few million up to $1 billion. In aggregate, approximately $2.7 billion was invested in convertible mortgages by the eight respondents who held this type of mortgage. Seventy-five percent of those investors stated that they have been satisfied with their investment to date. Twenty-five percent stated it was too early to say and none responded that they were dissatisfied.

Section III--Representative Features of Convertible Mortgages

Questions in this portion of the survey were intended to help determine what features are typical in a convertible mortgage agreement and on what type of property the mortgages normally are written. Table 11 displays the percentage of convertible mortgages invested in various types of real estate for each institutional investor. Their responses suggest that office buildings and shopping centers are the preferred areas of investment interests.

Table 11. Type of Property Collateralizing Convertible Mortgages.

Type of Property	Percentage invested in each property type by investors (1-8)							
	1	2	3	4	5	6	7	8*
Apartments			99.5					
Office Buildings	100	60	.5		70	40	60	
Shopping Centers		35		100	30	20	25	
Industrial		5				40	5	
Hotel/Motel							10	
Other								

*Investor 8 did not respond to this question.

The remainder of Section III relates to loan characteristics and the cash flows annually associated with a convertible mortgage. Questions 2-6 deal with the fixed income portion of the loan. Questions 7 and 8 provide information on participation income that often flows to the investor. The last seven questions provide insight into how the conversion privilege is structured in a typical convertible mortgage. Each of the cash flows is analyzed separately.

Interest Income

When asked how many basis points below a conventional, fixed-rate mortgage a convertible would be discounted, answers ranged from 100 to 350 basis points. Most respondents gave a range and suggested that the coupon discount rate varied, depending upon participation of income

from operations. Most loans are interest only (62%), with 13 percent being amortized, and 25 percent paying interest only until the conversion date and then starting to amortize if conversion does not occur. The typical loan maturity when written ranges between 10-20 years.

Loan-to-value ratios tended to be high relative to conventional, fixed-rate mortgages. The lowest ratio was 70 percent, while one investor said that it is possible to make loans for 100 percent of the property value. The majority of responses suggested that 80-90 percent ratios are typical. Investors' requirements varied greatly with regard to debt coverage ratios. Two respondents were willing to originate convertibles with debt coverage ratios as low as 100 percent. Half of the investors felt that 110 percent was typical. A ratio of 120 percent was the highest debt coverage cited. One investor felt the concept of debt coverage is archaic when evaluating convertible mortgages.

Participation income

Quite often convertible mortgages allow lenders (investors) to participate in income from operations. The survey suggests that 87 percent of the convertibles the investors were originating allow for some type of participation beyond interest income. Investors were asked what participation is based upon. Thirty-eight percent answered that participation is based upon cash flow after regular

debt service payments had been paid. Other responses said participation was based upon

- Gross collected
- Effective gross income
- Net operating income
- Preferred return to investor with participation income starting after preferred return is paid

Conversion right

The most interesting aspect of convertible mortgages is the privilege given the investor to convert a debt instrument into a position of ownership. Information gathered by this questionnaire provides some insight into how the conversion right is typically structured. Questions 9-15 of Section III focus on the conversion feature.

Twenty-five percent of the respondents say the conversion right first occurs in the fifth year. Another 25 percent stated that the tenth year is normally the earliest date of conversion. The remaining investors responded with ranges of 3-10, 7-9, 7-11, and 8-15 years to indicate when first conversion may occur. Investors' statements vary considerably as to whether conversion can occur more than once. Two investors say that the conversion privilege is offered only once. Another respondent stated that conversion could occur at the end of the tenth year or during the fifteenth year. The remaining investors differ on the date conversion first can occur, but say the right to convert is normally continuous after the first chance, if 9-12 months' notice is given the developer.

It does appear that once conversion occurs most contracts allow the lender to buy the developer's equity interest. Seventy-five percent of the responses indicate this is the case. When asked how the investor's (lender's) interest in the property is defined upon conversion, 37 percent responded that this interest is equal to the original loan-to-value ratio. Other responses included

- Negotiated at origination, frequently loan-to-value ratio
- Negotiated at origination, typically 50 percent
- Typically fifty percent interest (all have been second mortgages)--receive priority return on equity after conversion to the extent their equity exceeds that of developer
- Negotiated equity interest--after conversion lender assumes preferred position with cumulative preferred return on this account equal to the coupon rate of loan.

Regardless of how the lender's equity interest is determined, most property values at conversion are determined by independent appraisal. Five of the eight respondents (62.5%) state that an appraisal determines property value. One investor values the real estate via a formula agreed upon when the mortgage was written. Another assumes the property will be sold. The value is calculated as sale price less original cost less selling expenses. One investor says his firm does not value the property at conversion: it merely takes its preferred equity position. It is assumed that in this arrangement the lender does not automatically buy out the developer's position; rather, they become partners.

The survey also attempted to ascertain what growth (appreciation) prospects were normally associated with convertible mortgages. Survey question twelve asked how much annual appreciation is expected on a representative property. The next question requested an estimate of expected annual growth in the property's net operating income. Obviously these two rates are closely related, as property value is a function of the income-producing ability of a property. Answers to these questions were almost always identical. Growth rates for net operating income averaged 6.6 percent, while the average expected appreciation rate was 6 percent. The range in both cases was between 4-15 percent. The mode for both estimates was 5 percent.

How is a representative convertible mortgage structured? Section III sheds some light on this matter, but obviously no standard set of characteristics is common to all convertibles. This type of mortgage is unique and highly complex. It is not surprising that each mortgage might vary according to the needs of the parties at interest.

Some generalizations can be made regarding the typical convertible mortgage. The reader is cautioned, however, to remember the small sample set from which these data are drawn. Normally the projects' convertibles are written on either office buildings or shopping centers. Coupon rates

may be discounted 100-300 basis points below conventional, fixed-rate mortgages. Loans usually require interest payments only, not amortization. Loan-to-value ratios range from 80-100 percent. Most contracts call for some form of investor participation in operating income after debt service payments. The first opportunity for conversion is typically after the fifth year, and usually the conversion privilege occurs more than once. Upon conversion the investor normally has the right to buy out the developer's equity interest at the property's appraised value. Most originators of convertible mortgages have forecasted growth rates of net operating income and property appreciation rates to be approximately 5-6 percent. A relevant range for most characteristic features of convertible mortgages can be defined. There is not, however, a standard contract representative of all convertibles being written.

Section IV--Pricing Convertible Mortgages

Convertible mortgages are sophisticated hybrid securities which are quite difficult to price. Earlier work in this paper has attempted to develop models which might be used to price convertibles. How do the largest institutional investors determine an appropriate price at origination? Do they use complex computer assisted models or a more intuitive approach? Section IV of the questionnaire asked institutional investors this question.

In the survey the following statement was made:

> Briefly describe how your firm decides to price convertible mortgages. Specifically, how do you decide how to discount coupon rates a certain number of basis points below conventional, fixed-rate mortgages? Is the decision based upon a formula or perhaps a more intuitive approach?

Brief descriptions of the responses made by investors were

- Intuitive approach used--as it relates to each particular seller, property, market, etc.
- Convertibles must be priced 250-300 basis points below conventional loans to be accepted--however, our firm must be able to achieve our targeted 7.5 percent real rate of return.
- Intuitive approach--our firm needs an internal rate of return that is competitive on a risk-adjusted basis to other investment alternatives.
- Our firm attempts to get 200 basis points above the standard 30 percent fixed-rate mortgage return after sharing conversion profits.
- Pricing reflects level of risk being assumed on a particular property in a given market.
- Our firm tries to build a risk premium over the 10-year treasury bond rate.
- Our firm reviews the fixed income markets as well as the real estate market. Pricing is also subject to how much of a preferred return we are to receive. Each loan is priced differently subject to location, type of property, and general market conditions. Pricing generally falls at 100-150 basis points below fixed income markets.

Apparently institutional investors do not use complex models to evaluate convertible mortgage loans. They probably forecast cash flows based upon expected results. The estimated cash flows are then discounted to their present values to decide whether the loans are prudent. None of the respondents mentioned option pricing theory as a

way of pricing convertibles. Their approach is more intuitive, and this is understandable, given the number of factors which impact upon their investment decision. If convertible loans were more standardized, a more sophisticated approach might be employed.

Section V--Case Histories of Actual Convertible Mortgages

This portion of the questionnaire attempted to gather specific information on actual convertible mortgages written by institutional investors. Four mortgage contracts were provided by the respondents. These data, in conjunction with information in Section III, are used in Chapter Six to test the valuation models developed previously.

Summary

This chapter summarizes results of a questionnaire designed to gather data on investment in convertible mortgages. The survey was mailed to the fifty largest institutional investors in real estate. This group of professional investors has real estate holdings in excess of $200 billion. Thirty-six of the fifty questionnaires were returned. Twenty-eight (78 percent) of the respondents do not invest in convertibles. These investors did, however, provide useful information with regard to why they do not make this type of investment. Eight respondents originated convertible mortgages, and these investors furnished useful

data on this type of mortgage. They stated why they invest in convertibles, and how they structured the mortgages.

It is apparent from survey results that convertible mortgages are unique hybrid instruments which have not gained widespread acceptance as part of institutional investment portfolios. The complexity of their streams of income make these securities difficult to evaluate and price. Traditional financial analysis techniques do not appear totally satisfactory in valuation of these mortgages.

CHAPTER SIX
COMPARISON OF MODELS

Introduction

This chapter compares the models developed to value convertible mortgages. Insufficient data were derived from the survey to test the models' ability to approximate convertible mortgage prices. The survey did, however, provide enough information to identify the representative characteristics of these instruments.

These were used to develop a typical convertible mortgage against which the models were tested. The test is not of the models' correctness in valuing convertible mortgages, but rather, it demonstrates how the models estimate the value of a mortgage. By comparing results of the output of both models run on the same mortgage one can see what the models can and cannot accomplish, how they are similar, and how they are dissimilar.

Application and Comparison of the Models: A Case Example

Representative Convertible Mortgage

The representative mortgage is assumed to amount to $10,000,000 which represents 80 percent of a property's

initial appraised value ($12,500,000). The interest rate is 10.5 percent and only interest is paid until maturity in twenty years. The first option to convert to an equity position occurs in year five, at which time the lender can take an 80 percent equity position in the property. A second conversion option occurs in year ten. The loan also provides the lender with participation income equal to $200,000 per year for twenty years. A summary of the basic terms of the convertible mortgage is provided in Table 12.

Table 12. Loan terms.

Loan Amount	$10,000,000
Loan to Value Ratio	.80
Interest Rate	.105
Loan Maturity Date	20 years
Conversion Date	5 years
Conversion Share	.80

Model Input Variables

Tables 13 and 14 show the input parameters required for the Discounted Cash Flow (DCF) and Option Pricing Theory (OPT) models.

Cash flows associated with interest income and participation income are equal in both models, although the method of inputting the variable is quite different. However, the models do not incorporate expected future property value in the same way. The DCF model requires a forecast of future property value based upon either capitalization of future net operating income or compounding the original

Table 13. Parameters for DCF model.

Variable	Value
Initial Project Value	$12,500,000
Loan to Value Ratio	.80
Coupon Rate	.105
Required Rate of Return (interest income)	.125
Required Rate of Return (participation income)	.14
Required Rate of Return (conversion income)	.14
Loan Maturity Date	20 years
Conversion Date	5 years
Growth Rate of NOI	0
Appreciation Rate	.5
Standard Deviation of Appreciation Rate	.025
Participation Rate	.80

Table 14. Parameters for OPT Model.

Variable	Value
Value of Property, V	$12,500,000
Face Value of Debt, D	$10,000,000
Time Until Conversion, T	5 years
Participation Yield, δ	.02
Standard Deviation, σ	.10
Participation/Conversion Rate, α	.80
Riskless Rate, r	.09

property value at an assumed appreciation rate. When Monte Carlo simulation is used to generate a future value, both the mean growth (or appreciation) rate and the standard deviation growth rate are required inputs. In contrast, the OPT model does not require a forecast of future property value, but rather an estimate of volatility of the property's value is needed. Even though the standard deviation of future property value is common to both models, the forecast values may be quite different. This point

will be reviewed more thoroughly when the output of the models is compared.

One other major difference exists in required input and should be noted. The DCF Model requires an estimate of required rate of return for interest income, participation income, and conversion income. The OPT Model requires an estimate of the risk free interest rate over the period of the option.

Output Values

A comparison of the models will be made on four important output values. These outputs are the value of the entire convertible mortgage (CM), the straight debt portion of the instrument (D), participation income (P), and the conversion option (C). Each of these values is an estimate of the present worth of each cash flow element of a convertible mortgage. Table 15 portrays the output from both models based upon the representative convertible mortgage described earlier.

Table 15. Convertible mortgage valuation.

DCF MODEL:	Output	Value
	Value of Convertible Mortgage	$11,064,359
	Value of Interest Income	8,932,283
	Value of Participation Income	686,616
	Value of Conversion Option	1,445,460
OPT MODEL:	Output	Value
	Value of Convertible Mortgage	$11,163,907
	Value of Interest Income	9,884,213
	Value of Participation Income	951,626
	Value of Conversion Option	328,068

Output Interpretation

The DCF Model values the mortgage at $11,064,359 whereas the OPT Model values the convertible at $11,163,907. A difference in value of less than 1 percent. In contrast the values of the component parts (D,P,C) are very different between models and require explanation. The value differences for the straight debt and participation elements are tied directly to input variable assumptions. In the DCF Model required rates of return were specified and used as discount factors to determine the present worth of interest and participation income. The OPT Model also required an interest rate, but that rate was the riskless rate. If the relationship between the risk-free rate of return and required rate of return is not appropriately defined, the models will value the cash flows differently. Also the portion of the Black-Scholes formula which deals with property value volatility and the normal distribution function causes differences between the model values for C.

The OPT Model estimates future property value by allowing the estimated variance of the property value to be approximated by the normal distribution function. No specific value forecast must be made by the model user. In contrast, the DCF Model requires a forecast of the property appreciation rate. Any positive appreciation rate estimate will cause the option element of the DCF Model to be higher than that of the OPT Model.

Tables 16 and 17 contain a sensitivity analysis on both models. Sensitivity analysis provides insight as to how the four values respond to changes in input values. As would be expected, increases in the estimated appreciation rate in the DCF Model generate higher CM values. The value

Table 16. Convertible mortgage valuation-sensitivity analysis.

DCF Model	CM	D	P	C
Variable	Output			
Appreciation Rate, σ				
.0, 10	$ 9,653,300	$ 8,932,283	$ 686,616	$ 34,401
.0, .25	9,157,523	8,932,283	686,616	38,624
.05, .0125	11,068,430	8,932,283	686,616	1,449,531
.05, .025 (std.)	11,064,359	8,932,283	686,616	1,445,460
.05, .05	11,037,862	8,932,283	686,616	1,418,963
.10, .05	12,764,627	8,932,283	686,616	3,145,728
Conversion Year				
5 (std.)	11,064,359	8,932,283	686,616	1,445,460
10	11,250,318	8,510,690	1,043,223	1,696,405
OPT Model				
Variable				
Standard Deviation (σ)				
.05	$11,316,443	$10,318,788	$ 951,626	$ 46,030
.10 (std.)	11,163,907	9,884,213	951,626	328,068
.15	11,070,904	9,419,926	951,626	699,352
Riskless Rate				
.08	11,357,843	10,178,700	951,626	227,517
.09 (std.)	11,163,907	9,884,213	951,626	328,068
.10	10,970,797	9,561,894	951,626	457,277
Conversion Year				
5 (std.)	11,163,907	9,884,213	951,626	328,068
10	10,262,299	6,589,667	1,812,692	1,859,939

Table 17. Sensitivity analysis dynamics.

As Variable Increases	Affect on Value* CM	D	P	C
DCF Model:				
Appreciation Rate	+	/	/	+
σ-sigma	-	/	/	-
Time Until Conversion	+	-	+	+
OPT Model:				
σ-sigma	-	-	/	+
Riskless Rate	-	-	/	+
Time Until Conversion	-	-	+	+

* +, value increases
-, value declines
/, value is unchanged

of the mortgage increases because the value of the conversion option rises with higher property value estimates. Increasing the variance associated with the appreciation rate has exactly the opposite effect in the DCF Model as the standard deviation of the assumed appreciation rate rises, the option element declines in value which reduces the value of CM. The OPT Model responds differently to changes in the variance of property values. As the variance increases, so does the value of the option portion of the mortgage. This phenomenon would hold true for any option priced by the Black-Scholes model. The increased risk impacts negatively on the debt element. This occurs because default risk increases as the variance rate rises.

When the time-until-maturity is increased, all component elements in both models move in the same direction; however, the value of the total package increases

in the DCF Model while the value falls in the OPT Model. This occurs because the debt element will fall since the loan had a discounted coupon rate and increasing loan maturity merely lengthens the time required to recoup a discounted interest rate loan. The participation income increase is strictly a function of additional proceeds being received over the added years.

The option element also increases in value for both models. The increase in the DCF Model arises from the fact that a favorable appreciation estimate continues to compound the estimated future property value. The OPT Model responds favorably to the longer time until conversion because the added time increases the likelihood that the property value will fluctuate over a wider range of values. The reason the overall effect is positive in the DCF Model is that the expected property appreciation more than offsets the lost interest on the discounted loan. In the OPT Model the increases in participation income and conversion value evidently do not offset the loss opportunity costs associated with the debt element.

Summary

This chapter compares the models developed in this study to evaluate convertible mortgages. The comparison was made based upon analysis of a representative convertible mortgage. An attempt was made to structure the model inputs in a manner which would create similar expected cash flows. Interest income and participation income cash flows were

exactly the same for both models. The manner in which the models were constructed did not allow the estimated future property value to be precisely the same. Even if all cash flows could have been perfectly correlated, the required rates of return in the DCF Model and the riskless rate in the OPT Model might have caused some bias. If these variable values are not correctly related, a fair comparison of the models would not occur.

With the understanding that there may have been some difference in the estimated future property value and that interest rate bias may have existed the models were applied. The results suggest that the models value convertible mortgages similarly with regard to overall value. Values of the component elements of the mortgage were, however, not very similar. Reasons for the variation in estimated values of interest income, participation income and conversion value income were discussed.

The mathematics in the DCF Model are quite simple and the logic of the formula is straightforward. If this model is to be profitably used, however, care must be taken to use plausible input values. Particular attention should be given to choosing appropriate required rates of return, growth rates for net operating income, and the estimated appreciation rates associated with future property value. If the user extends the model by incorporating the Monte Carlo simulation feature and applying sensitivity analysis, the usefulness of the discounted cash flow technique is

greatly enhanced. These model extensions allow the user to study probability distributions of output values to focus on feasible ranges of possible outcomes. Both the Monte Carlo simulation and sensitivity analysis features generate better data for decision-making.

The OPT Model is mathematically more sophisticated than the DCF Model. Even though the formula is complicated, its major attraction is that most of the required input variables are observable. Only five variables are required, and four of these (time-to-maturity, appraised property value, risk free rate, and the exercise price) are fairly easy to determine. The other variable (volatility) is more difficult to determine, but means for doing so exist. Another promising aspect of the OPT Model is that it does not depend on any assessment of the future or expected property price. The model makes no assumptions regarding the investors' assessments of the probabilities that the property will go up or down. Also, it does not depend on investors' attitudes toward risk. The fact that the option formula is independent of expectations and other subjective measures bodes well for its applicability.

CHAPTER SEVEN
CONCLUSION

The primary purpose of this paper is to develop valuation methods for convertible mortgages. Discounted cash flow analysis and option pricing theory are used to estimate the value of the conversion right in a representative convertible mortgage. The formulas developed using these techniques are computer programmed and utilized to incorporate risk analysis into the decision-making process.

The appropriateness of these models could not be evaluated on an empirical basis due to lack of data. Consequently the evaluation was carried out by an analysis of the important features or characteristics of the convertible mortgage. Particularly Chapter 4, shows how, why, or why not to apply option pricing theory to the evaluation of convertibles.

A survey of large institutional real estate investors was conducted to ascertain how and why convertible mortgages are currently being used. Survey responses provide insight into the industry's perception of convertibles, the common features of the instruments being written, and an indication of how investors are currently valuing these instruments. The data generated by the questionnaire provided the basic

information necessary to construct a representative convertible mortgage. The valuation models developed in the study also depend upon survey data for determination of appropriate cash flow sources.

Extensive risk analysis of a representative convertible is also performed. The DCF Model incorporated both Monte Carlo simulation and sensitivity analysis. The OPT formula inherently embodied risk analysis by considering the instantaneous variance of the property value and employing the normal distribution function. The OPT Model was also subjected to sensitivity analysis.

Chapter 6 compares the output generated by the two models. Each of the primary component elements of a convertible mortgage was valued. Differences in the values produced by the two models were noted and analyzed. The OPT Model appears to have considerable promise in terms of practical usefulness because its input variables are generally observable. The OPT Model does not require the user's subjective opinion about future property value estimates and it does not depend on investors' attitudes toward risk. If users incorporate the Monte Carlo simulation and sensitivity analysis features of the DCF Model, some of the subjectivity associated with that model's inputs can either be eliminated or at least brought into focus.

No decision was made regarding which model is superior. It is believed that either model can assist investors in making better investment decisions. The models might be

used by investors who desire a more accurate approach to pricing convertible mortgages. But to suggest that either of the models correctly prices convertible mortgages would be premature. Further research is needed to determine how well the models work on actual convertibles. There appear to be few actual convertible mortgages and there is no marketplace where they are traded. Perhaps as a proxy of convertible mortgages, convertible bonds could be tested. Continued research in this area also could proceed if institutional investors who originate convertible mortgages are willing to cooperate. The response to the questionnaire and personal contact with institutional investors suggest that this possibility does exist.

APPENDIX I
CONVERTIBLE MORTGAGE VALUATION COMPUTER MODEL

```
FILE: IFPS     IFPS    A1  NORTHEAST REGIONAL DATA CENTER VM/SP RELEASE 3 CMS

READY FOR MODELING LANGUAGE COMMAND
INPUT:  LIST
1
MODEL CMVM VERSION OF 09/30/85 19:42
 10 COLUMNS 1-&HP&
 20 HP 1ESS 1=&HP&-1
 30 INITIAL PROJECT VALUE=10000000
 40 LTV RATIO=.8
 50 COUPON RATE=.105
 60 RORINT=.135
 70 RORPAR=.15
 80 RORCV=.15
 90 LOAN LIFE=20
 100 CONVERSION DATE=&HP&
 110 CAPITALIZATION RATE=.1
 120 GROWTH RATE=NORRANDR (.05,.005)
 130 NET OPERATING INCOME=924000,PREVIOUS*(1+GROWTH RATE)
 140 LOAN AMOUNT=0 FOR HP LESS 1,INITIAL PROJECT VALUE*LTV RATIO
 150 INTEREST PAYMENT=INITIAL PROJECT VALUE*LTV RATIO*COUPON RATE
 160 CONVERSION VALUE=0 FOR HP LESS1, NET OPERATING INCOME*LTV RATIO/'
 165    CAPITALIZATION RATE
 170 PARTICIPATION RATE=.80
 180 PARTICIPATION INCOMWE=(NET OPERATING INCOME-INTEREST PAYMENT)*'
 185    PARTICIPATION RATE
 190 CASH IN1=INTEREST PAYMENT + PARTICIPATION INCOME + CONVERSION VALUE
 200 VALUE OF CONVERTIBLE MORTGAGE=NPVC(INTEREST PAYMENT,RORINT,0)'
 205    + NPVC(PARTICIPATION INCOME,RORPAR,0) + NPVC(CONVERSION VALUE, RORCV,0)
 210 VALUE OF CONVERSION OPTION='
 211    NPVC(INTEREST PAYMEN,RORINT,0)+'
 213    NPVC(PARTICIPATION INCOME,RORPAR,0)'
 215    +NPVC(CONVERSION VALUE,RORCV,0)-NPVC(INTEREST PAYMENT,RORINT,0)'
 217    -NPVC(PARTICIPATION INCOME,RORPAR,0)-NPVC(LOAN AMOUNT,RORCV,0)
 220 INTEREST FOREGONE=INITIAL PROJECT VALUE*LTV RATIO*(RORINT-COUPON RATE)
 230 COST OF OPTION=NPVC(INTEREST FOREGONE, RORINT, 0)-'
 235    NPVC(PARTICIPATION INCOME,RORPAR,0)
 240 CASH OUT3=LTV RATIO*INITIAL PROJECT VALUE,0 FOR HP LESS 1
 250 LENDER IRR=IRR(CASH 1N1,CASH OUT 3)
 260 FORECASTED ANNUAL PROPERTY APPRECIATION=IRR(CONVERSION VALUE,CASH OUT3)
 270 BREAKEVEN CONVERSION VALUE=NTV(INTEREST FOREGONE,RORINT,0)'
 275    -NTV(PARTICIPATION INCOME,RORPAR,0)+LOAN AMOUNT
 278 CASH IN3=0 FOR HP LESS 1,BREAKEVEN CONVERSION VALUE
 280 ANNUAL PROPERTY APPRECIATION REQUIRED TO BREAKEVEN='
 285    IRR(CASH IN3,CASH OUT3)
```

```
295    (INITIAL PROJECT VALUE*LTV RATIO)
300 OPTION VALUE RATIO=L210/L230
310 NET OPTION VALUE=L210-L230
END OF MODEL
INPUT: SOLVE
ENTER DEFINITION FOR: HP
INPUT: 5
MODEL CMVM VERSION OF 09/30/85 19:42 -- 5 COLUMNS 31 VARIABLES
ENTER SOLVE OPTIONS
INPUT: ALL
```

APPENDIX II
OPTION PRICING THEORY

Convertible Mortgage Valuation Computer Model

```
FILE; CM      IFPS      A1  NORTHEAST REGIONAL DATA CENTER

FILE     FORMULA ND      05/08/8617:13:17
10 INPUTS X8
11 X9=IF X8.GE.0 THEN X8 ELSE -1*X8
20 K=1/(1+P3*X9)
30 P3=.33267
40 A1=.4361836
50 A2=(-1)*.1201676
60 A3=.9372980
70 PI=3.14
80 N1=(1/XPOWERY(2*PI,.5))*NATEXP(-1*X9*X9/2)
90 N2=1-N1*(A1*K+A2*K*K+A3*K*K*K)
100 ND=IF X8.GE.0 THEN N2 ELSE 1-N2
FILE      MODEL     CM     05/08/8616:33:12
1 ACCESS FORMULA ND,NDP
10 V=1000
20 X=1288
30 SIGMA=.025
40 T=5
50 P1=.02
60 P2=.8
70 R=.05
80 VAR=SIGMA*SIGMA
90 X5=SIGMA*XPOWERY(T,.5)
100 X1=(-1*NATLOG(V/X)-(R-P1+(VAR/2))*T)/X5
110 X2=(NATLOG(V/X) + (R-P1-(VAR/2))*T)/X5
120 X3=(NATLOG(P2*V/X) + (R-P1+VAR/2)*T)/X5
130 X4=(NATLOG(P2*V/X) + (R-P1-VAR/2)*T)/X5
140 P=P2*V*(1-NATEXP(-1*P1*T))
150 D=V*NATEXP(-1*P1*T)*ND(X1)+NATEXP(-1*R*T)*X*ND(X2)
160 C=P2*V*NATEXP(-1*P1*T)*ND(X3)-NATEXP(-1*R*T)*X*ND(X4)
170 CM=D+P+C
180 R1=ND(X1)
190 R1=ND(X2)
200 R3=ND(X3)
210 R4=ND(X4)
220 GEORGE=NATLOG(V/X)
230 XRAY=T-P1+VAR/2
```

APPENDIX III
CONVERTIBLE MORTGAGE QUESTIONNAIRE

Introduction

This questionnaire is an attempt to find the current practice of investment in convertible mortgages. Your assistance is greatly appreciated and results from this research will be available for your use after analysis of the collected date is complete. Naturally, you can be assured of confidentiality since your firm's identity is never recorded.

The questionnaire consists of five parts which gather the following data:

1. Who invests in convertible mortgages?
2. When and why does a firm invest in convertible mortgages?
3. What are the representative characteristics of convertible mortgages?
4. How do investors price convertible mortgages?
5. What are the specifics of an actual convertible mortgage your firm has invested in?

A better understanding of how and why convertible mortgages are being used should be available after collecting and analyzing the data generated by this questionnaire. We hope that this research will provide

important insights into the industry's perception of convertibles, the common features of the instruments being written, and an indication of how the appropriate coupon discount is currently being established.

CONVERTIBLE MORTGAGE QUESTIONNAIRE

(Please circle the correct answer.)

SECTION I--Background Information

1. How would you categorize your firm?

   A. Commercial Bank
   B. Mortgage Banker
   C. Mortgage Broker
   D. Savings & Loan
   E. Pension Fund
   F. Insurance Company
   G. REIT
   H. Other, please specify _____

2. In what state is your firm's home office located?

3. What is the approximate value of your real estate portfolio? (mortgages and equity real estate; but not mortgage-backed securities such as Ginnie Mae pass-throughs and Freddie Macs)

4. What is the approximate value of your income property real estate portfolio? (mortgages and equity)

5. Does your firm invest in convertible mortgages?

   A. Yes
   B. No

6. If your firm does not invest in convertible mortgages, why? (circle all that are appropriate)

   A. Question legal ramifications
   B. Do not believe they are an attractive investment
   D. Other, please specify _____

7. If your firm does not invest in convertible mortgages, do you expect to do so within the next two years?

   A. Yes
   B. No
   C. Uncertain

SECTION II--Convertible Mortgage Commitments

Note: Please continue questionnaire only if your firm invests in convertible mortgages.

1. If your firm does invest in convertible mortgages, why? (circle all that are appropriate)

   A. Viewed as effective inflation hedge
   B. Favorable returns based on risk taken
   C. Provide investment diversification
   D. Other, please specify _____

2. Under what circumstances would your firm invest in a convertible mortgage but not a conventional, fixed-rate mortgage?

3. Does your firm view convertible mortgages as more or less risky than conventional, fixed-rate mortgages?

   A. More Risky
   B. Less Risky

      Why?

4. If your firm does invest in convertible mortgages; what is the dollar amount invested in convertible mortgages?

   $_____

5. If your firm does invest in convertible mortgages, have you been satisfied with these investments to date?

   A. Yes
   B. No
   C. Too early to say

SECTION III--Representative Features of Convertible Mortgages Your Firm Writes

1. Approximately what percent of your convertible mortgages are in

   A. Apartments _____percent
   B. Office Buildings _____percent
   C. Shopping Centers _____percent
   D. Industrial _____percent
   E. Hotel/Motel _____percent
   F. Other, please specify _____percent

   100% total

2. What is the discount in interest rate charges on convertible mortgages as compared to conventional, fixed-rate mortgages?

   A. 100 basis points
   B. 150 basis points
   C. 200 basis points
   D. 250 basis points
   E. 300 basis points
   F. 350 basis points
   G. 400 basis points
   H. Other, please specify _____

3. Are loans amortized or interest only?

   A. Amortized
   B. Interest only

4. What is the typical loan maturity when written?

   A. 10 years
   B. 15 years
   C. 20 years
   D. 25 years
   E. Other, please specify _____

5. What is the loan to value ratio

   A. 50 percent
   B. 60 percent
   C. 70 percent
   D. 80 percent
   E. 90 percent
   F. Other, please specify _____

6. What is the debt coverage ratio?

   A. 110 percent
   B. 120 percent
   C. 130 percent
   D. Other, please specify _____

7. Do these loans normally allow for cash flow participation?

   A. Yes
   B. No.

8. If the loans normally are participation loans, what is the participation based upon?

   A. Effective gross income
   B. Net operating income
   C. Cash flow after regular debt service payments
   D. Other, please specify _____

9. When does the first conversion option occur?

   A. Fifth year
   B. Seventh year
   C. Tenth year
   D. Other, please specify _____

10. If the conversion option occurs more than once, when does it start and what are the intervals between options?

    Year it first occurs; _____year
    Intervals occur how frequently: _____

11. Upon conversion does your firm have the option to buy the developer's equity interest?

    A. Yes
    B. No

12. How much annual appreciation is expected on a representative property?

    _____percent per year

13. How much annual growth is expected in the property's net operating income?

    _____percent per year

14. Upon conversion how is your firm's interest in the property defined?

    A. Lender's interest is equal to the original loan to value ratio.
    B. Other, please specify.

15. Upon conversion how is the value of the property determined?

SECTION IV--Pricing of Convertible Mortgages

1. Briefly describe how your firm decides to price convertible mortgages. Specifically, how do you decide how to discount coupon rates a certain number of basis points below conventional, fixed-rate mortgages? Is the decision based upon a formula or perhaps a more intuitive approach?

SECTION V--Case History of an Actual Convertible Mortgages

(These data will be used to test a recently developed formula for valuation of convertible mortgages.)

Please provide the following information regarding an actual representative convertible mortgage investment your firm has made. If it is more convenient, mail a photocopy of an actual convertible mortgage agreement and provide any information below not found in the contract.

Loan amount _____

Loan to value ratio _____

Coupon rate on convertible mortgage _____

Approximate discount in basis points below conventional fixed-rate mortgage _____

Original maturity of loan _____

Amortized or interest only? _____

Type of property _____

Location of property (state) _____

Conversion date(s) _____

Option to buy out developer? _____

If yes, how is value decided? _____

Participation in cash flows? _____

If yes, how? _____

Has loan been converted? _____

If yes, how many years after origination? _____

If loan has been converted, what was the approximate value of your interest upon conversion? _____

How is (was) the property's value determined at conversion?

What were your firm's expectations with regard to annual growth in:

Net operating income (for example at what growth rate were pro forma cash flows projected at the time the loan was made) _____

Property appreciation _____

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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